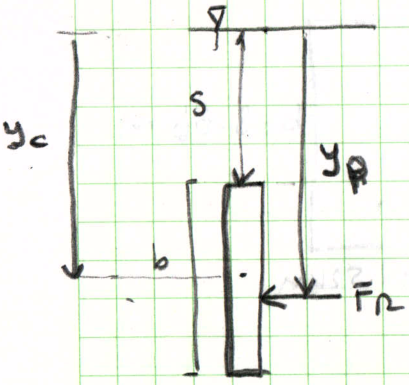
Örnek 2.12: Araba Sorusu



$$F_R = P_c \cdot A = \gamma_{su}\, y_c \underbrace{(a \cdot b)}_{\text{Alan}}$$

$$F_R = (\rho_{su}\, g)\left(\frac{b}{2}+s\right)(a \cdot b)$$

$$F_R = \left(1000\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{1.2}{2}+8\right)(1.2 \times 1)$$

$$\boxed{F_R = 101239\ N = 101.24\ kN}$$

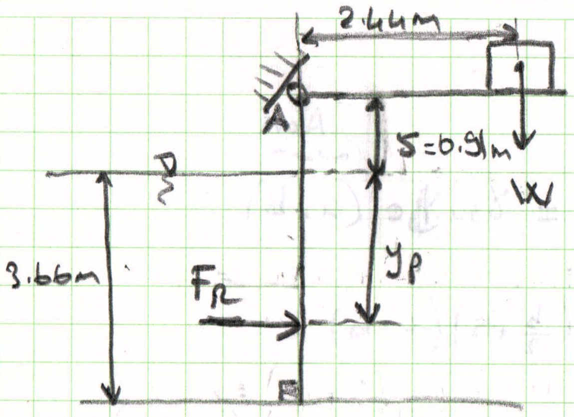
Yaklaşık 10 ton'luk bir kuvvete karşılık gelir.

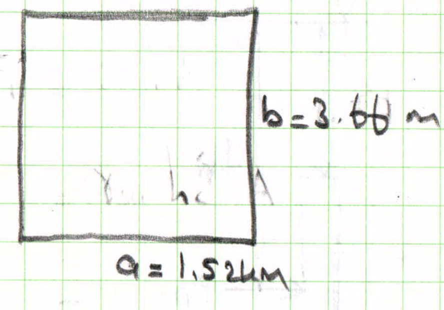
$$y_p = y_c + \frac{I_{xx,c}}{y_c \cdot A} =$$

$$y_p = \left(\frac{b}{2}+s\right) + \frac{\cancel{a}\, b^{\cancel{3}\,2}/12}{\left(\frac{b}{2}+s\right)(\cancel{a \cdot b})}$$

$$y_p = \left(\frac{1.2}{2}+8\right) + \frac{(1.2)^2/12}{\left(\frac{1.2}{2}+8\right)} \Rightarrow \boxed{y_p = 8.61\ m}$$

Örnek 3.13: L Kapak





$F_R = P_c . A$

$= \gamma_{su} . h_c . (a \times b)$

$= \rho_{su}\, g \left(\frac{b}{2}\right) (a \times b)$

$= \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(\frac{3.66}{2} m\right)(1.524 \times 3.66\ m^2)$

$F_R = 100135\ N$

$y_p = y_c + \frac{I_{xx,c}}{y_c A}$

$y_p = \frac{b}{2} + \frac{ab^3/12}{\frac{b}{2}(a \times b)}$

$y_p = \frac{2b}{3} = \frac{2.(3.66)}{3}$

$\boxed{y_p = 2.44\ m}$

$\Sigma M_A = 0$

9.81 m/s²

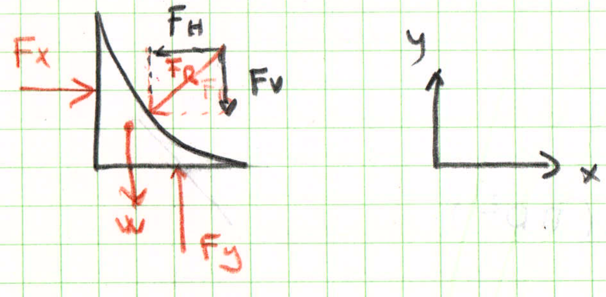
$W.(2.44) = F_R(y_p + s)$

$mg\,(2.44) = F_R(2.44 + 0.91)$

$m = \frac{F_R}{(9.81)} \frac{3.35}{2.44}$

$\boxed{m = 14014\ kg}$

Örnek 3.15



• $\Sigma F_x = 0 \Rightarrow \underline{F_x = F_H}$

$$F_x = P_c A$$

$$F_x = (\gamma_{su} h_c) A = (\rho_{su} g h_c) A$$

$$F_x = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(4.2 + \frac{0.8}{2}\right)(0.8m \times 1m)$$

$$\boxed{F_x = 36100\ N = 36.1\ kN = F_H}$$

$F_R = 52.3\ kN$

$\theta = 46.4^\circ$

• $\Sigma F_y = 0 \Rightarrow F_y = W + F_v$

$$F_v = F_y - W = 39.2\ kN - 1.3\ kN \Rightarrow \boxed{F_v = 37.9\ kN}$$

$$F_y = (P_c \cdot A)$$

$$F_y = (\gamma_{su} h_c A)$$

$$F_y = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(5m)(0.8m \times 1m)$$

$$\underline{F_y = 39240\ N} = 39.2\ kN$$

$$W = mg = \rho_{su} g V = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(R^2 - \frac{\pi R^2}{4}\right)(1m)$$

$$\underline{W = 1.3\ kN}$$

Omek 3.16

3-75 A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at B where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to A at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** The atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

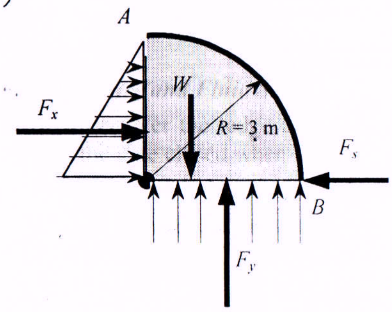
Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g(R/2)A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(3/2\text{ m})(4\text{ m}\times 3\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 176.6\text{ kN}$$



Vertical force on horizontal surface (upward):

$$F_y = P_{ave}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(3\text{ m})(4\text{ m}\times 3\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 353.2\text{ kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g V = \rho g[w\times \pi R^2/4]$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)[(4\text{ m})\pi(3\text{ m})^2/4]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 277.4\text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 353.2 - 277.4 = 75.8\text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(176.6\text{ kN})^2 + (75.8\text{ kN})^2} = 192.2\text{ kN}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{75.8\text{ kN}}{176.6\text{ kN}} = 0.429 \rightarrow \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 192.2 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point A where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad \rightarrow \quad F_R R\sin(90-\theta) - F_{\text{spring}} R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R \sin(90-\theta) = (192.2\text{ kN})\sin(90° - 23.2°) = \mathbf{177\text{ kN}}$$

örnek 3.17

$$F_R = P_c\, A$$

$$F_R = \gamma_{su}\, h_c\, (\pi D^2)$$

$$F_R = \gamma_{su}\left(H - \frac{D}{2} \cdot \sin 50\right)(\pi D^2)$$

$$25\ N = (\rho_{su}\, g)(H - 0.02 \sin 50)(\pi\, 0.04^2)$$

$$25\ N = \left(1000\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(H - 0.02 \sin 50)(\pi \cdot 0.04^2)$$

$$H = 0.522\ m$$

$$\gamma_{cıva} \cdot h = \gamma_{su} \cdot (H + 0.02)$$

$$h = \frac{\gamma_{su}}{\gamma_{cıva}}(H + 0.02) = \frac{1000\ (kg/m^3)}{13600\ kg/m^3}(0.522 + 0.02)$$

$$\boxed{h = 0.04\ m = 4\ cm}$$

Örnek 5.1

a) $\forall = 3m \times 1.8m \times 1.8m = 9.72\ m^3$

$$Q = \frac{\forall}{\Delta t} = \frac{9.72\ m^3}{3\ dk} = \frac{9.72\ m^3}{3 \times 60\ s}$$

$$Q = 0.054\ m^3/s$$

b)

$$\cancel{\frac{\partial}{\partial t}\int \rho\, d\forall} + \underbrace{\rho_2 V_2 A_2}_{\text{çıkan}} - \underbrace{\rho_1 V_1 A_1}_{\text{giren}} = 0$$

sürekli akış

$$\cancel{\rho_1} V_1 A_1 = \cancel{\rho_2} V_2 A_2 \qquad \rho_1 = \rho_2 \quad : \text{Sıkıştırılamaz akış}$$

sıkıştırılamaz akış

$$V_1 A_1 = V_2 A_2 = Q \quad (m^3/s)$$

$$V_1 = \frac{Q}{A_1} = \frac{0.054\ m^3/s}{\pi (0.02)^2/4} \Rightarrow \boxed{V_1 = V_{giren} = 1.72\ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.054\ m^3/s}{\pi (0.01)^2/4} \Rightarrow \boxed{V_2 = V_{çıkan} = 6.87\ m/s}$$

Örnek 5.2

$$\cancel{\frac{\partial}{\partial t}\int \rho\, d\forall} + \rho_2 V_2 A_2 - \rho_1 V_1 A_1 = 0$$

sürekli

$$\cancel{\rho_1} V_1 A_1 = \cancel{\rho_2} V_2 A_2$$

sıkıştırılamaz akış $\rho_1 = \rho_2$

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

$$\bar{V}_2 = \frac{\int \cancel{\rho} V_2\, dA_2}{\cancel{\rho} A_2}$$

$$U_1\ A_1 = A_2$$

$$\boxed{\bar{V}_1 = U = \bar{V}_2}$$

K.H'nın tam kesitinde ortalama hız U'dur.

$$dA_2 = 2\pi r\, dr$$

$$\bar{V}_2 = \frac{\int V_2\, 2\pi r\, dr}{A_2}$$

$$\bar{V}_2 = \frac{\int u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r\, dr}{\pi R^2}$$

$$= \frac{2\cancel{\pi}\, u_{max}}{\cancel{\pi} R^2} \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$= \frac{2u_{max}}{R^2}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\Bigg|_0^R$$

$$= \frac{2u_{max}}{R^2}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) = \frac{2\, u_{max}}{\cancel{R^2}} \frac{\cancel{R^2}}{4}$$

$$\boxed{\bar{V}_2 = \frac{u_{max}}{2} = U}$$

Örnek 5.3

a) $$\frac{\partial}{\partial t}\int \rho\, d\forall + \rho_2 V_2 A_2 - \rho_1 V_1 A_1 = 0$$

çıkan kütle yok

$$\frac{\partial}{\partial t}\int \rho\, d\forall = \rho_1 V_1 A_1$$

$\forall = Ah$

$\frac{d\forall}{dh} = A$

$d\forall = A\,dh$

$$\cancel{\rho}\frac{\partial}{\partial t}\int d\forall = \cancel{\rho_1} V_1 A_1$$

$$\frac{\partial}{\partial t}\int A\,dh = V_1 A_1 = Q = 0.24\ \frac{m^3}{dk}$$

$$A\frac{\partial h}{\partial t} = Q$$

$$\frac{\partial h}{\partial t} = \frac{Q}{A} = \frac{0.24\ m^3/dk}{3m \times 1.5m} \Rightarrow \boxed{\frac{\partial h}{\partial t} = 0.053\ \frac{m}{dk}}$$

b) $$\frac{dh}{dt} = 0.053$$

$$\int_{h=0}^{h=1.8} dh = \int_{t=0}^{t} 0.053\, dt$$

$$h\Big|_0^{1.8} = 0.053t\Big|_0^t$$

$$1.8 = 0.053t$$

$$\boxed{t \cong 34\ dk}$$

örnek 5.4

$$\frac{\partial}{\partial t}\int \rho \, d\forall + \rho_2 V_2 A_2 - \cancel{\rho_1 V_1 A_1} = 0$$

Giriş kütle yok

$$\frac{\partial}{\partial t}\int \cancel{\rho}\, d\forall = -\cancel{\rho_2} V_2 A_2 \rightarrow \rho = \rho_1 = \rho_2 \text{ sıkıştırılamaz akış}$$

$$\frac{\partial}{\partial t}\int d\forall = -V_2 A_2 \qquad \forall = A_T h \rightarrow \frac{d\forall}{dh} = A_T \rightarrow d\forall = A_T dh$$

$$\frac{\partial}{\partial t}\int A_T\, dh = -V_2 A_2$$

$$\frac{\cancel{\pi} D_{tank}^2}{\cancel{4}} \frac{\partial h}{\partial t} = -V_2 \frac{\cancel{\pi} D_{jet}^2}{\cancel{4}}$$

$$\frac{\partial h}{\partial t} = -\sqrt{2gh}\,\frac{D_{jet}^2}{D_{tank}^2}$$

$$\frac{1}{\sqrt{h}}\, dh = -\sqrt{2g}\left(\frac{D_{jet}}{D_{tank}}\right)^2 dt$$

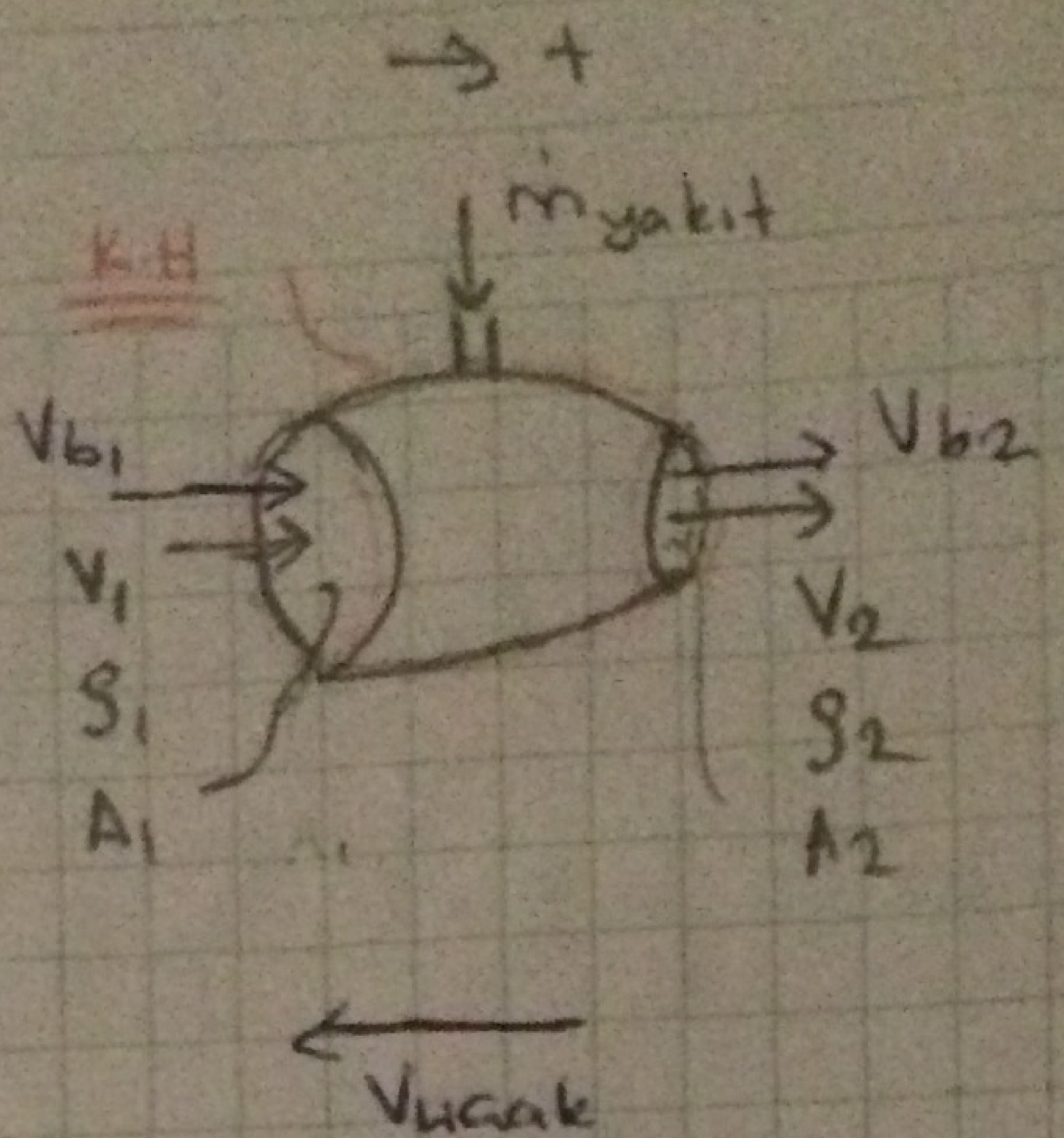
$$\int_{h_0=1.219}^{h=0.609} \frac{1}{\sqrt{h}}\, dh = \int_0^t -8.55\times10^{-4}\, dt$$

$$2\sqrt{h}\,\Big|_{1.219}^{0.609} = -8.55\times10^{-4}\, t\,\Big|_0^t$$

$$2\left(\sqrt{0.609} - \sqrt{1.219}\right) = -8.55\cdot10^{-4}(t-0)$$

$$\boxed{t = 757 \text{ s} \approx 12.6 \text{ dk}}$$

Örnek 5.5



← $V_{uçak} = 971 \ km/h$

$A_1 = 0.8 \ m^2$ $\qquad \rho_1 = 0.736 \ kg/m^3$

$V_2 = 1050 \ km/h$

$A_2 = 0.558 \ m^2$

$\rho_2 = 0.515 \ kg/m^3$

* Akış sürekli ve sıkıştırılabilir çünkü giriş ve çıkış kesitleri arasında yoğunluk farkı var.

$$\cancel{\frac{\partial}{\partial t}\int_{KH} \rho \, d\forall} + \sum \dot{m}_{çıkan} - \sum \dot{m}_{giren} = 0$$

sürekli akış

$$\sum \dot{m}_{giren} = \sum \dot{m}_{çıkan}$$

$$\dot{m}_{yakıt} + \dot{m}_1 = \dot{m}_2$$

$$\dot{m}_{yakıt} = \dot{m}_2 - \dot{m}_1$$

$$\dot{m}_{yakıt} = \rho_2 V_{b2} A_2 - \rho_1 V_{b1} A_1$$

* $V_{b1} = V_1 - V_{KH}$

$$\boxed{V_{b1} = V_1 = V_{uçak} = 971 \ km/h}$$

uçak motoru giriş yönündeki hız henüz kontrol hacmine girmediği için direkt olarak uçağın hızına eşit olmalıdır

* $V_{b2} = V_2 - V_{KH}$

$$V_{b2} = 1050 \ \frac{km}{h} - \left(-971 \ \frac{km}{h}\right)$$

$$V_{b2} = 2021 \ km/h$$

$$V_{b2} = 2021 \times 10^3 \ m/h$$

$$\dot{m}_{yakıt} = \left(0.515 \ \frac{kg}{m^3}\right)\left(2021 \times 10^3 \ \frac{m}{h}\right)\left(0.558 \ m^2\right)$$

$$- \left(0.736 \ \frac{kg}{m^3}\right)\left(971 \times 10^3 \ \frac{m}{h}\right)\left(0.8 \ m^2\right)$$

$$\boxed{\dot{m}_{yakıt} = 9050 \ kg/h}$$

Örnek 5.6

Zamana bağlı, sıkıştırılamaz akış.

K.H → sabit (hareketsiz), şekil değiştiriyor

$$\frac{\partial}{\partial t}\int_{KH} \cancel{\rho}\, dV + \sum(\rho \underbrace{V A}_{Q})_{çıkış} - \sum(\rho VA)_{giriş} = 0$$

sıkıştırılamaz akış

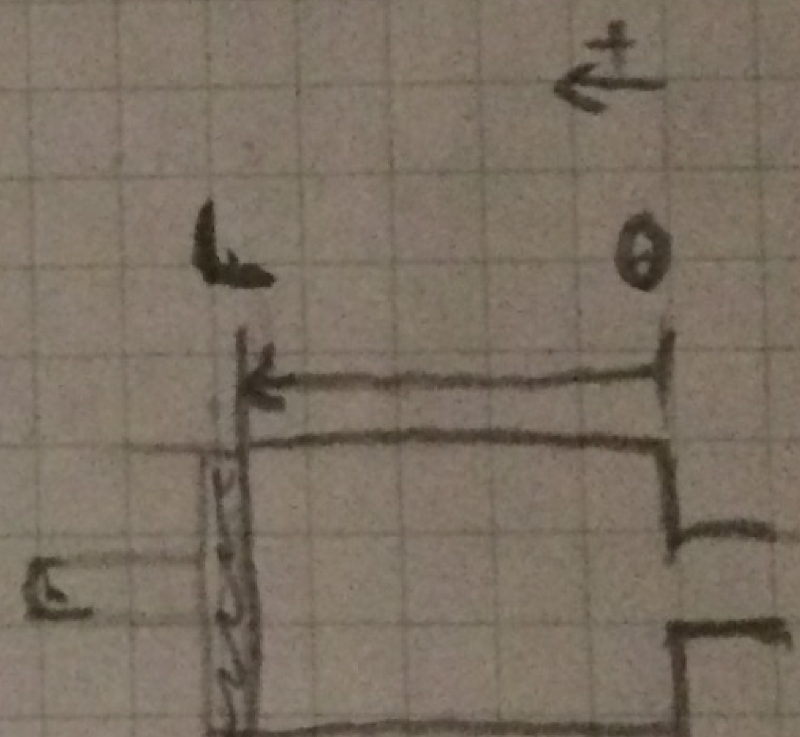
giriş yok

$$\frac{\partial}{\partial t}\int \cancel{\rho}\, dV = -\sum(\rho V A)_{çıkış}$$

sıkıştırılamaz akış → Q

$$\frac{\partial}{\partial t}\int dV = -(\overbrace{-V_2 A_2}^{Q_2} + \overbrace{V_s . A}^{Q_s = Q_2 \cdot 0.1}) = -(-0.9\, Q_2) = 0.9\, Q_2$$

$$V = A_p L \Rightarrow \frac{dV}{dL} = A_p \Rightarrow dV = A_p\, dL$$



$$\frac{\partial}{\partial t}\int A_p\, dL = 0.9\, Q_2$$

$$A_p \frac{\partial}{\partial t}\int_L^0 dL = 0.9\, Q_2$$

$$-\frac{\partial L}{\partial t} = \frac{0.9\, Q_2}{A_p} = \frac{0.9\,(300\times 10^3\ mm^3/dk)}{500\ mm^2}$$

$$\boxed{\frac{\partial L}{\partial t} = -54\ \frac{mm}{dk}} = V_p$$

$$\boxed{V_p = -54\ \frac{mm}{dk} = 54\ \frac{mm}{dk}\ (\rightarrow)}$$

54 ve kanalın ağırlığı ihmal

Basınç kuvvetleri ihmal

örnek 5.7

$$\sum \vec{F} = \frac{\partial}{\partial t}\cancel{\int_{KH} \vec{V} \rho \, d\forall} + \sum (\vec{V} \rho V A)_{çıkış} - \sum (\vec{V} \rho V A)_{giriş}$$

sürekli akış

Bu eşitliğin x ve y yönlerinde iki bileşeni vardır.

$$\rightarrow \sum F_x = \sum (V_x \rho V A)_{çıkış} - \sum (V_x \rho V A)_{giriş}$$

$$F_{Ax} = V_{2x} \rho_2 V_2 A_2 - V_{1x} \rho_1 V_1 A_1$$

Sıkıştırılamaz daimi tek akış için kütlenin korunum denklemi $\rightarrow \dot{m}_1 = \dot{m}_2$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad (A_1 = A_2 = A)$$

$$V_1 = V_2 = 3\,m/s \qquad (\rho_1 = \rho_2 = \rho)$$

$$F_{Ax} = V_2 \cdot \cos\theta \, \rho V_2 \cdot A - V_1 \rho V_1 A \qquad (V_1 = V_2)$$

$$= V_1^2 \rho A (\cos\theta - 1)$$

$$= (3\,m/s)^2 \left(1000 \frac{kg}{m^3}\right)(0.006\,m^2)(\cos\theta - 1)$$

$$\boxed{F_{Ax} = 54(\cos\theta - 1)} \; N$$

$$\rightarrow \sum F_y = V_{2y} \rho_2 V_2 A_2 - \underbrace{\cancel{V_{1y}}}_{0} \rho_1 V_1 A_1$$

$$F_{Ay} = V_2 \cdot \sin\theta \cdot \rho_2 V_2 A_2$$

$$F_{Ay} = (3\,m/s)(\sin\theta)(1000\,kg/m^3)(3\,m/s)(0.006\,m^2)$$

$$\boxed{F_{Ay} = 54 \sin\theta} \; N$$

$$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} \rightarrow$$

Örnek 5-8

$$\sum \vec{F} = \underbrace{\cancel{\frac{\partial}{\partial t}\int_{KH} \vec{V} \rho \, d\forall}}_{\text{Sabit akış}} + \sum (\vec{V}\dot{m})_{çıkış} - \sum (\vec{V}\dot{m})_{giriş}$$

(*) $\xrightarrow{+} \sum F_x = \sum (\underbrace{\cancel{V_x \dot{m}}}_{\text{çıkış -y yönünde}})_{çıkış} - \sum (V_x \dot{m})_{giriş}$

$$R_x + P_1 \cdot A_1 = - V_1 \cdot \dot{m}_{giriş}$$

$$P_{1,e} = P_{1,mutlak} - P_{atm}$$

$$R_x = -P_{1e} A_1 - V_1 \cdot \dot{m}_{giriş}$$

$$R_x = -\left((220-101)\times 10^3 \frac{N}{m^2}\right) \cdot (0.01\ m^2) - \left(4 \frac{m}{s}\right)\left(40 \frac{kg}{s}\right)$$

$$R_x = -1350\ N$$

$$\boxed{R_x = 1350\ N\ (\leftarrow)}$$

(*) Sıkıştırılamaz daimi akış için toplam kütlenin korunumu (süreklilik) denklemi

$$\dot{m}_{giren} = \dot{m}_{çıkan}$$

$$\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} = (16\ m/s) \frac{0.0025\ m^2}{0.01\ m^2}$$

$$\underline{V_1 = 4\ m/s}$$

$$\dot{m}_{giriş} = \rho V_1 A_1 = \left(1000 \frac{kg}{m^3}\right)\left(4 \frac{m}{s}\right)\left(0.01\ m^2\right)$$

$$\underline{\dot{m}_{giriş} = 40\ kg/s}$$

(*)

$\uparrow \sum F_y = \sum (V_y \dot{m})_{çıkış} - \sum (\underbrace{\cancel{V_y \dot{m}}}_{\text{giriş x yönünde}})_{giriş}$

$$\underbrace{-\cancel{W}}_{\text{ihmal edelim}} + R_y = -V_2\, \dot{m}_{çıkış}$$

↳ V_2 hızının yönünden dolayı

$$R_y = -\left(16 \frac{m}{s}\right)\left(40 \frac{kg}{s}\right)$$

$$R_y = -640\ N$$

$$\boxed{R_y = 640\ N\ (\downarrow)}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{1350^2 + 640^2}$$

$$\boxed{R = 1494\ N}$$

Örnek 5.5

$$\sum \vec{F} = \cancel{\frac{\partial}{\partial t}\int_{KH} \vec{V}\rho\, dV} + \sum(\vec{V}\dot{m})_{çıkan} - \sum(\vec{V}\dot{m})_{giren}$$

Daimi akış

$\dot{m}_{giren} = \dot{m}_{çıkan} = Q\cdot\rho = 110\ \frac{kg}{s}$

$\overset{+}{\rightarrow} \sum F_x = V_{2x}\dot{m}_{çıkan} - V_1\dot{m}_{giren}$

$R_x + (P_1 - P_{atm})A_1 - (P_2 - P_{atm})A_2 \cdot \cos 30 = V_2 \cdot \cos 30\ \dot{m}_{çıkan} - V_1 \cdot \dot{m}_{giren}$

- $Q_1 = 0.11\ \frac{m^3}{s} = V_1 \cdot A_1 \Rightarrow V_1 = \frac{Q_1}{A_1} = \frac{0.11\ m^3/s}{0.0182\ m^2} = 6.044\ m/s$
- $Q_1 = Q_2 = V_2 \cdot A_2 \Rightarrow V_2 = \frac{Q_2}{A_2} = \frac{0.11\ m^3/s}{0.0081\ m^2} = 13.580\ m/s$

$R_x + (200-101)\times 10^3\ \frac{N}{m^2}(0.0182\ m^2) - (120-101)\times 10^3\ \frac{N}{m^2}(0.0081\ m^2)\cos 30$

$= 110\ \frac{kg}{s}\left(13.580\cos 30 - 6.044\ \frac{m}{s}\right)$

$R_x + 1801.8\ \frac{N}{m^2} - 133.281\ \frac{N}{m^2} = 628.829\ \frac{N}{m^2}$

$R_x = -1039.69\ N \Rightarrow \boxed{R_x = 1039.69\ N\ (\leftarrow)}$

$+\uparrow \sum F_y = V_{2y}\dot{m}_{çıkan} - \cancel{V_{1y}\dot{m}_{giren}}\ 0$

hızın yönünden

$R_y - W_{su} - W_{dirsek} + (P_2 - P_{atm})A_2 \cdot \sin 30 = -V_2 \cdot \sin 30\ \dot{m}_{çıkan}$

$R_y - V\rho g - M_{dirsek}\, g + (120-101)\times 10^3\ \frac{N}{m^2}(0.0081\ m^2)\sin 30 = -(13.580\ \frac{m}{s})(\sin 30)\, 110\ \frac{kg}{s}$

$R_y - 58.86 - 98.1 + 76.95 = -746.9$

$R_y = -666.89 \Rightarrow \boxed{R_y = 668.89\ (\downarrow)}$

örnek 5.10

Sürekli ve bir boyutlu akış (yalnızca z yönde akış giriş bunu) halinde yalnızca z-yönde bağlantı kuvveti ortaya çıkaracaktır.

$$\uparrow \sum F_z = \sum(V_z \dot m)_{çıkış} - \sum(V_z \dot m)_{giriş}$$

etkin basınç — etkin basınç — Serbest jet $\boxed{P_2 = 0}$

$$* \; +\uparrow \sum F_z = F_A - W_{kole} - P_1 A_1 - W_{su} + \cancel{P_2 A_2}\,0$$

$$\uparrow * \; \sum(V_z \dot m)_{çıkış} = V_2 \cdot \dot m_a \;\rightarrow\; \dot m_a = Q_a \rho = \left(0.6\times10^{-3}\,\frac{m^3}{s}\right)\left(1000\,\frac{kg}{m^3}\right)$$

$$\boxed{\dot m_a = 0.6\ kg/s}$$

$$\sum(V_z \dot m)_{çıkış} = 0.6\,V_2$$

$$V_2 = \frac{Q_a}{A_a} = \frac{0.6\times10^{-3}\ m^3/s}{\pi(5\times10^{-3})^2/4\ m^2} = 30.6\ m/s$$

$$\sum(V_z \dot m)_{çıkış} = -0.6\,V_2$$

hızın yönü aşağı doğru (−z) yönde

$$* \; \sum(V_z \dot m)_{giriş} = V_1 \cdot \dot m_g \qquad \dot m_g = \dot m_a = 0.6\ kg/s$$

→ Sürekli akış için kütlenin korunum denklemi

$$\sum(V_z \dot m)_{giriş} = 0.6\,V_1 \qquad Q_g = Q_a = 0.6\times10^{-3}\ m^3/s$$

$$\sum(V_z \dot m)_{giriş} = -0.6\,V_1\ N \qquad V_1 = \frac{Q_g}{A_g} = \frac{(0.6\times10^{-3}\ m^3/s)}{\pi(16\times10^{-3})^2/4\ m^2} = 2.98\ m/s$$

$$F_A - \underbrace{(0.1\,kg)\left(9.81\,\frac{m}{s^2}\right)}_{0.981\,N} - \overbrace{\left(464\times10^3\,\frac{N}{m^2}\right)\left(\frac{\pi(16\times10^{-3})^2}{4}\,m^2\right)}^{93.29\,N} - \overbrace{(0.003\,kg)\left(9.81\,\frac{m}{s^2}\right)}^{0.029\,N}$$

$$= \sum(V_z \dot m)_{çıkış} - \sum(V_z \dot m)_{giriş}$$

$$F_A - 0.981\,N - 93.29\,N - 0.029\,N = \dot m_a(-V_2) - \dot m_g(-V_1)$$

$$F_A - 94.3\,N = 0.6(V_1 - V_2) = 0.6(2.98 - 30.6)\ N$$

$$\boxed{F_A = 77.73\ N}$$

Örnek 5.11

K.H: İdeal gibi akan boruyu ve lendeki suyu içeriyor.

$$(+)\rightarrow \quad \sum F_y = \sum (V_y \dot{m})_{çıkan} - \sum (V_y \dot{m})_{giren}$$

$$F_{Ay} + P_1 A_1 + P_2 A_2 = (-V_2 \dot{m}_ç) - (V_1 \dot{m}_g) \qquad \dot{m}_ç = \dot{m}_g = \dot{m}$$

$$F_{Ay} + \left((207 - 101.3) \times 10^3 \frac{N}{m^2}\right)(0.01\, m^2) + \left((165 - 101.3) \times 10^3 \frac{N}{m^2}\right)(0.01\, m^2)$$

$$= -\dot{m}(V_2 + V_1)$$

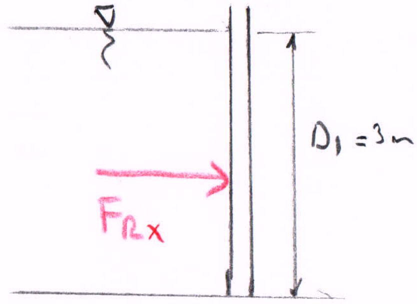
$$\dot{m} = \rho_1 A_1 V_1 = 150\ kg/s$$

$$F_{Ay} + 1057 + 637 = -\left(150 \frac{kg}{s}\right)(15 + 15)\frac{m}{s}$$

$$F_{Ay} = -6194\ N \Rightarrow \boxed{F_{Ay} = 6194\ N\ (\leftarrow)}$$

Örnek 5.12

a) Kapak kapalı olduğunda yatay kuvvet



w : Kanalın genişliği

$\boxed{w = 1\ m \text{ kabul edelim}}$

$$F_{Rx} = P_c . A$$

$$F_{Rx} = \gamma_{su} . h_c . (D_1 . w)$$

$$F_{Rx} = (\rho_{su} g)\, h_c . (D_1 . w)$$

$$F_{Rx} = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(\frac{3}{2} m\right)(3m \times 1m) \Rightarrow \boxed{F_{Rx} = 44145\ N = 44.145\ kN}$$

b) Daimi akış

$$\sum \vec{F} = \cancel{\frac{\partial}{\partial t} \int_{KH} \vec{V} \rho\, d\forall} + \sum (\vec{V} \dot{m})_{çıkış} - \sum (\vec{V} \dot{m})_{giriş}$$

$$\xrightarrow{+} \sum F_x = (V_x \dot{m})_{çıkış} - (V_x \dot{m})_{giriş}$$

sürtünme kuvvetini ihmal edelim

$$R_x + F_{R1} - F_{R2} - \cancel{F_f} = V_2 . \dot{m}_{çıkış} - V_1 \dot{m}_{giriş}$$

$$R_x + (\rho_{su} g\, h_{c1})(D_1 w) - (\rho_{su} g\, h_{c2})(D_2 w) = V_2 \dot{m}_{çıkış} - V_1 \dot{m}_{giriş}$$

Daimi sıkıştırılamaz akış kabulü

$\dot{m}_{giriş} = \dot{m}_{çıkış} = \dot{m}$

$$R_x + (\rho_{su} g\, h_{c1})(D_1 w) - (\rho_{su} g\, h_{c2})(D_2 w) = \dot{m}\,(V_2 - V_1)$$

$$R_x + \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(\frac{3m}{2}\right)(3m \times 1m) - \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(\frac{0.429 m}{2}\right)(0.429\ m \times 1m) =$$

$$\left(3000 \frac{kg}{s}\right)\left(7 \frac{m}{s} - 1 \frac{m}{s}\right)$$

$$R_x + 44145\ N - 902.72\ N = 18000\ N$$

$$R_x = -25242.28\ N = -25.24\ kN$$

$$\boxed{R_x = 25.24\ kN \leftarrow}$$

Örnek 5.13

Uydu sabit V_0 hızı ile hareket etmektedir. Bu nedenle eğer referans koordinat sistemi uydu üzerindeki herhangi bir nokta da alınabilir. Böylece akışkan hızları sadece hareketli uyduya göre olan bağıl hızlar halini alır.

$$\sum \vec{F} = \frac{\partial}{\partial t}\int \vec{V}\rho\, dV + \sum(\vec{V}\dot{m})_{çıkış} - \sum(\vec{V}\dot{m})_{giriş}$$

$$\cancel{\sum \vec{F}} = \frac{d(m\vec{V})_{KH}}{dt} + \sum(\vec{V}\dot{m})_{çıkış} - \cancel{\sum(\vec{V}\dot{m})_{giriş}}$$

Uyduya etkiyen herhangi bir dış kuvvet yoktur.

K.H giren bir kütle yoktur.

Kabul: Atılan yakıtın kütlesi uydunun kütlesinin yanında ihmal edilebilir. Bu nedenle m_{uydu} sabit kabul edilebilir.

Kabul: Yanma odasından olan gaz akışı bir boyutludur.

$$(\overset{+}{\rightarrow})\quad m_{uydu}\frac{dV_{KH}}{dt} = -\sum(V_x\dot{m})_{çıkış}$$

$$\dot{m}_{çıkış} = \frac{100\ kg}{2\ s} = 50\ \frac{kg}{s}$$

$$m_{uydu}\frac{dV_{KH}}{dt} = -\left(-V_f\,\dot{m}_{çıkış}\right)$$

$$m_{uydu}\frac{dV_{KH}}{dt} = V_f\,\dot{m}_{çıkış}$$

$$\frac{dV_{KH}}{dt} = \frac{V_f\cdot\dot{m}_{çıkış}}{m_{uydu}} = \frac{(3000\ m/s)(50\ kg/s)}{5000\ kg} = 30\ m/s^2$$

a) $\boxed{a_{uydu} = \frac{dV_{KH}}{dt} = 30\ m/s^2}$

b) $\int_0^V dV_{KH} = \int_0^2 30\,dt$

$$V_{KH}\Big|_0^V = 30\cdot t\Big|_0^2$$

$$\boxed{V_{KH} = 30\cdot 2 = 60\ m/s}$$

c) $F = m\cdot a$

$$F = (5000\ kg)(30\ m/s^2)$$

$$F = 150000\ N = 150\ kN$$

örnek 5.15

$V_2=0$

$$\frac{P_1}{\rho_{su}} + \frac{V_1^2}{2} + \cancel{g z_1} = \frac{P_2}{\rho_{su}} + \cancel{\frac{V_2^2}{2}} + \cancel{g z_2} \qquad (z_1 = z_2)$$

$$\frac{V_1^2}{2} = \frac{P_2 - P_1}{\rho_{su}}$$

$$V_1 = \sqrt{\frac{2(P_2-P_1)}{\rho_{su}}} \longrightarrow P_1 = \gamma_{su}(h_1+h_2) \quad P_2 = \gamma_{su}(h_1+h_2+h_3)$$

$$V_1 = \sqrt{\frac{2}{\rho_{su}} \gamma_{su}(h_1+h_2) - \gamma_{su}(h_1+h_2+h_3)}$$

$$V_1 = \sqrt{\frac{2}{\rho_{su}} \gamma_{su} (\cancel{h_1+h_2} - \cancel{h_1} - \cancel{h_2} - h_3)}$$

$$V_1 = \sqrt{\frac{2}{\cancel{\rho_{su}}} \cancel{\rho_{su}}\, g\, h_3}$$

$$V_1 = \sqrt{2 g h_3} = \sqrt{2(9.81 m/s^2)(0.12 m)}$$

$$\boxed{V_1 = 1.53 \ m/s}$$

Örnek 5.16

$$P_{2,e} = P_2 - P_{atm} = P_{atm} - P_{atm} = 0$$

$$\frac{P_{1,e}}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \cancel{\frac{P_{2,e}}{\rho}} + \frac{V_2^2}{2} + \cancel{g z_2} \qquad \underline{z_1 = z_2}$$

$$P_{1,e} = (P_1 - P_{atm}) = \frac{\rho(V_2^2 - V_1^2)}{2} = \frac{(1.23\ kg/m^3)(50^2 - 10^2)}{2} = 1476\ Pa = 1.48\ kPa$$

* V_1 hızını bulurken ise süreklilik denklemi kullanırız.

$$\dot{m}_1 = \dot{m}_2$$

$$\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} = \left(50\ \frac{m}{s}\right) \frac{0.02\ m^2}{0.1\ m^2}$$

$$\underline{V_1 = 10\ m/s}$$

Örnek 5.17

a) ② noktasındaki hızı yani serbest jet hızını belirlemek için ① ve ② noktaları arasına Bernoulli Denklemi uygulayalım.

$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + g z_1 = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + g z_2 \qquad (P_1 = P_2 = P_{atm})$$

$V_1 \approx 0$

$$\dot m_1 = \dot m_2$$

$$\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} \qquad A_2 \ll A_1 \rightarrow V_1 \approx 0 \text{ alınabilir.}$$

$$\frac{V_2^2}{2} = g(z_1 - z_2)$$

$$V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2 \cdot (9.81\,m/s^2)(0-(-7))}$$

$$\boxed{V_2 = 11.72\ m/s}$$

b) $$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}} + g z_1 = \frac{P_A}{\rho} + \frac{V_A^2}{2} + g z_A \qquad V_1 \approx 0$$

Boru kesiti sabit

$V_A = V_2$

$\dot m_A = \dot m_2$

$\cancel{\rho} V_A \cancel{A} = \cancel{\rho} V_2 \cancel{A}$

$V_A = V_2$

$$P_A = P_1 - \rho \frac{V_A^2}{2} + \rho g (z_1 - z_A)$$

$$P_A = 101 \times 10^3 \frac{N}{m^2} - 1000 \frac{kg}{m^3} \frac{(11.72\ m/s)^2}{2} + \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(0 - 1\,m)$$

$$P_A = 22510.8 \frac{N}{m^2} = 22.51\ kPa \quad (\text{mutlak})$$

$$P_{A,e} = P_A - P_{atm} = 22.51 - 101 = -78.49\ kPa$$

Örnek 5.18

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}}^{0} + gz_1 = \cancel{\frac{P_3}{\rho}} + \frac{V_3^2}{2} + gz_3$$ $P_1 = P_3 = P_{atm}$

$$V_3 = \sqrt{2g(z_1 - z_3)} = \sqrt{2.(9.81)(0-(-6))}$$

$$\boxed{V_3 = 10.85 \text{ m/s} = V_2}$$

$$\frac{P_1}{\rho} + \cancel{\frac{V_1^2}{2}}^{0} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$P_2 = 1.765$ kPa (mutlak) (15°C 'de suyun buharlaşma basıncı)

$$gz_2 = \frac{P_1 - P_2}{\rho} - \frac{V_2^2}{2} + gz_1$$

$$z_2 = \frac{P_1 - P_2}{\rho g} - \frac{V_2^2}{2g} + z_1$$

$$z_2 = \frac{(101 \times 10^3 - 1.765 \times 10^3)\ N/m^2}{(1000\ \frac{kg}{m^3})(9.81\ \frac{m}{s^2})} - \frac{(10.85\ m/s)^2}{2.(9.81\ \frac{m}{s^2})} + 0$$

$z_2 = 4.12$ m

$z_2 = H - 4.5 = 4.12$ m $\Rightarrow$ $\boxed{H_{max} = 8.62 \text{ m}}$

Örnek 5.19

$V_1 \approx 0$

$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + gz_1 = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + gz_2 \qquad P_1 = P_2 = P_{atm}$$

$$V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2(9.81)(0.2-0)}$$

$V_2 = 1.98$ m/s

$$\dot{Q}_2 = V_2 \cdot A_2 = \left(1.98 \frac{m}{s}\right)\left(\frac{\pi (0.01 m)^2}{4}\right)$$

$$\dot{Q}_2 = 1.55 \times 10^{-4} \frac{m^3}{s}$$

Örnek 5.20

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Durma Noktası

$$P_1 + \cancel{\frac{\rho V_1^2}{2}} + \rho g z_1 = P_2 + \frac{\rho V_2^2}{2} + \rho g z_2$$

$$P_1 - P_2 + \rho g (z_1 - z_2) = \rho \frac{V_2^2}{2}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_p} + 2g(z_1 - z_2)}$$

ρ_p Spetrol

γ manometre sıvısı

$$P_1 + \gamma_p \cdot L = P_2 + \gamma_p (L + 1.5 - 0.2) + \gamma_m (0.2)$$

$$P_1 - P_2 = -\cancel{\gamma_p L} + \cancel{\gamma_p L} + \gamma_p (1.3) + \gamma_m (0.2)$$

$$P_1 - P_2 = \rho_p g (1.3) + \rho_m g (0.2) = (860)(9.81)(1.3) + (2500)(9.81)(0.2)$$

$$P_1 - P_2 = 15872.58 \left(\frac{N}{m^2} = Pa\right)$$

Örnek 5.20 (Devam)

$$V_2 = \sqrt{\frac{2 \cdot (P_1 - P_2)}{\rho} + 2g(z_1 - z_2)}$$

$$V_2 = \sqrt{\frac{2 \cdot (15872.58 \ N/m^2)}{860 \ kg/m^3} + 2 \cdot (9.81)(0 - 1.5)}$$

$$V_2 = \sqrt{7.48} \Rightarrow \boxed{V_2 = 2.74 \ m/s}$$

$$Q = V_2 \cdot A_2 = (2.74 \ m/s)(100 \times 10^{-4} \ m^2)$$

$$\boxed{Q = 0.0274 \ m^3/s}$$

Örnek 5.21 : Buhar Türbini : Mil İşi

$\dot{m}_g = \dot{m}_ç = \dot{m} \rightarrow$ Daimi, sıkıştırılamaz akış

$$\frac{\cancel{\dot{m}}\left(h_ç + \frac{V_ç^2}{2} + g z_ç\right) - \cancel{\dot{m}}\left(h_g + \frac{V_g^2}{2} + g z_g\right)}{\cancel{\dot{m}}} = \frac{\dot{W}_{mil,net}}{\dot{m}} \qquad \left(\frac{W}{kg/s}\right)$$

Kot farkı yok $\rightarrow z_ç = z_g$

$$(h_ç - h_g) + \left(\frac{V_ç^2 - V_g^2}{2}\right) + \cancel{g(z_ç - z_g)} = w_{mil,net} \qquad (J/kg)$$

Türbin iş yapar

$$(2550 - 3348)\times 10^3 \left(\frac{J}{kg}\right) + \frac{(60\,m/s)^2 - (30\,m/s)^2}{2} = -w_{mil,ç} + \cancel{w_{mil,g}} \qquad (J/kg)$$

$$-798\times 10^3 \left(\frac{J}{kg}\right) + 1350 \left(\frac{J}{kg}\right) = -w_{mil,ç}$$

$$\boxed{w_{mil,ç} = 796650 \ \frac{J}{kg} = 796.65 \ \frac{kJ}{kg}}$$

Örnek 5.22: Pompalara Geçiş

$$\dot{m}_a = \dot{m}_g = \dot{m} = \rho \dot{V} = 1000 \frac{kg}{m^3} \cdot 50\times10^{-3} \frac{m^3}{s} = 50 \ kg/s$$

$$\frac{\dot{m}\left(h_a + \frac{V_a^2}{2} + gz_a\right) - \dot{m}\left(h_g + \frac{V_g^2}{2} + gz_g\right)}{\dot{m}} = \frac{\dot{W}_{mil,net} + \dot{W}_{sınır,net}}{\dot{m}} \quad (W)$$

$$\left(h_a + \frac{V_a^2}{2} + gz_a\right) - \left(h_g + \frac{V_g^2}{2} + gz_g\right) = w_{mil,net} + w_{sınır,net} \quad (J/kg)$$

$$\left(u_a + \frac{P_a}{\rho} + \frac{V_a^2}{2} + gz_a\right) - \left(u_g + \frac{P_g}{\rho} + \frac{V_g^2}{2} + gz_g\right) = (w_{mil,g} - w_{mil,a}) + w_{sınır,net}$$

$$\frac{P_g}{\rho} + \frac{V_g^2}{2} + gz_g + w_{mil,g} = \frac{P_a}{\rho} + \frac{V_a^2}{2} + gz_a + w_{mil,a} + \underbrace{(u_a - u_g) + w_{sınır,net}}_{e_{kayıp}}$$

Kot farkı yok → $z_g = z_a = 0$

$$\frac{P_g}{\rho} + \cancel{\frac{V_g^2}{2}} + \cancel{gz_g} + w_{mil,g} = \frac{P_a}{\rho} + \cancel{\frac{V_a^2}{2}} + \cancel{gz_a} + \cancel{w_{mil,a}} + e_{kayıp} \quad (J/kg)$$

$(V_g = V_a)$

Giriş ve çıkış boru açıları eşit: $m_g = m_a$

$\rho V_g A = \rho V_a A$

$\boxed{V_g = V_a}$

$$\frac{P_g - P_a}{\rho} + w_{mil,g} = e_{kayıp} \quad (J/kg)$$

$$\frac{(100-300)\times10^3}{1000} + \frac{(0.9)(15\times10^3)}{50} = e_{kayıp}$$

$$e_{kayıp} = 70 \ \frac{J}{kg} \rightarrow \dot{E}_{kayıp} = \dot{m}\, e_{kayıp} = 50 \frac{kg}{s} \ 70 \frac{J}{kg}$$

$$\dot{E}_{kayıp} = 3500 \ W = 3.5 \ kW$$

$$\eta = \frac{\dot{W}_{mil} - \dot{E}_{kayıp}}{\dot{W}_{mil}} = \frac{13.5 - 3.5}{13.5} = 0.741 \rightarrow \%74.1$$

b) Kayıp olan bu enerji ısıya dönüşerek akışkanın sıcaklığını artırır

$$\dot{E}_{kayıp} = \dot{m} c_p \Delta T$$

$$\Delta T = \frac{\dot{E}_{kayıp}}{\dot{m} c_p} = \frac{(3509,5\ W)}{\left(50 \frac{kg}{s}\right) \cdot \left(4.18 \times 10^3 \frac{J}{kg K}\right)}$$

$$\boxed{\Delta T = 0.017\ K}$$

Örnek 5.23 : Hidrolik güç üretimi

$P_1 = P_2 \cong P_{atm}$ $\quad V_1 \sim 0$ $\quad V_2 \sim 0$

$$\cancel{\frac{P_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + g z_1 + \cancel{w_{mil,g}} = \cancel{\frac{P_2}{\rho}} + \cancel{\frac{V_2^2}{2}} + g z_2 + w_{mil,ç} + e_{kayıp} \quad (J/kg)$$

$$\frac{g z_1}{g} \Rightarrow \frac{g z_2 + w_{türbin} + e_{kayıp}}{g} \quad (J/kg)$$

$$(z_1 - z_2) = \underbrace{\frac{w_{türbin}}{g}}_{h_{türbin}} + \underbrace{\frac{e_{kayıp}}{g}}_{h_{kayıp}} \quad (m)$$

$$(z_1 - z_2) - h_{kayıp} = h_{türbin}$$

$$h_{türbin} = (120 - 0) - 35 \Rightarrow \underline{h_{türbin} = 85\ m}$$

$$\dot{W}_{türbin} = \dot{m}\, w_{türbin} = \dot{m} g h_{türbin} = (\rho \cdot \dot{V})\, g\, h_{türbin} \quad \rho = 1000$$

$$\dot{W}_{türbin} = \left(1000 \frac{kg}{m^3}\right)\left(100 \frac{m^3}{s}\right)\left(9.81 \frac{m}{s^2}\right)(85\ m)$$

$$\boxed{\dot{W}_{türbin} = 83.4\ MW}$$

$$\dot{W}_{elektrik} = \eta_{türbin} \cdot \dot{W}_{türbin} = (0.8)(83.4\ MW)$$

$$\boxed{\dot{W}_{elektrik} = 66.7\ MW}$$

örnek 5.24 : yük kayıpları ve güç kaybı

$g \to 1 \quad , \quad ç \to 2$

$$\cancel{\frac{P_g}{\rho}} + \cancel{\frac{V_g^2}{2}} + g z_g + w_{mil,g} = \cancel{\frac{P_ç}{\rho}} + \cancel{\frac{V_ç^2}{2}} + g z_ç + \cancel{w_{mil,ç}} + e_{kayıp}$$

$P_g = P_ç = P_{atm} \quad V_g \approx 0 \quad V_ç \approx 0$

$$g(z_g - z_ç) + w_{pompa} = e_{kayıp} \qquad (J/kg)$$

$$(z_g - z_ç) + \frac{w_{pompa}}{g} = \frac{e_{kayıp}}{g} = h_{kayıp} \qquad (m)$$

$$h_{kayıp} = (0 - 45\,m) + \frac{20 \times 10^3\ W}{(30\,kg/s)(9.81\,m/s^2)}$$

$$\dot{m} = \rho \dot{V} = 1000\,\frac{kg}{m^3} \cdot 0.03\,\frac{m^3}{s}$$

$$\dot{m} = 30\ kg/s$$

$$\boxed{h_{kayıp} \cong 23\ m}$$

$$\dot{E}_{kayıp} = \dot{m}\, e_{kayıp} = \dot{m}\, g\, h_{kayıp}$$

$$= \left(30\,\frac{kg}{s}\right)\left(9.81\,\frac{m}{s^2}\right)(23\,m)$$

$$\boxed{\dot{E}_{kayıp} \cong 6768.9\ W = 6.769\ kW}$$

örnek 5.25 : Fan işi ve verimi

$v_g = 0$ $\quad z_g = z_ç$

$$\cancel{\frac{P_g}{\rho}} + \cancel{\frac{V_g^2}{2}} + \cancel{g z_g} + w_{mil,g} = \cancel{\frac{P_ç}{\rho}} + \frac{V_ç^2}{2} + \cancel{g z_ç} + \cancel{w_{mil,ç}} + e_{kayıp} \quad (J/kg)$$

$P_g = P_ç = P_{atm}$

$$\dot{m}\left(w_{fan} - \frac{V_ç^2}{2}\right) = \left(e_{kayıp}\right)\dot{m} \quad \left(\frac{J}{kg} \cdot \frac{kg}{s}\right)$$

$\dot{m} = \rho A . V$

$\dot{m} = \left(1.23 \frac{kg}{m^3}\right)\left(\frac{\pi\, 0.6^2}{4} m^2\right)\left(12 \frac{m}{s^2}\right)$

$\dot{m} = 4.17 \ kg/s$

$$\dot{W}_{fan} - \frac{\dot{m} V_ç^2}{2} = \dot{E}_{kayıp} \quad (\dot{W} = J/s)$$

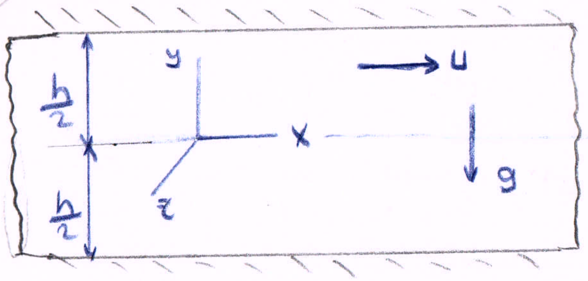
$$0.4 \times 10^3 \ W - \frac{(4.17 \ kg/s)(12 \ m/s)^2}{2} = \dot{E}_{kayıp}$$

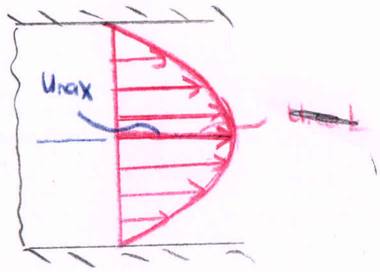
$$\boxed{\dot{E}_{kayıp} = 99.76 \ W \cong 0.1 \ kW}$$

- $\dot{E}_{faydalı} = \dot{W}_{fan} - \dot{E}_{kayıp} = 0.4 - 0.1 = 0.3 \ kW$

- $\eta = \dfrac{\dot{W}_{fan} - \dot{E}_{kayıp}}{\dot{W}_{fan}} = \dfrac{0.3}{0.4} \cong 0.75 \Rightarrow \%75$

Sabit Plakalar Arasında Daimi Laminer Akış (Düzlemsel Poiseuille Akışı)





Şekilde gösterilen sonsuz genişlikteki ~~ve uzunluktaki~~ dar bir boşlukta Newton tipi bir akışkana ait daimi, sıkıştırılamaz laminer akışı dikkate alalım,

Kabuller

1) Plakalar x- ve z- yönünde sonsuz.
2) Daimi akış
3) Paralel akış : Bu geometride akışkan parçacıkları plakalara paralel x-yönünde hareket etmektedir, y- ve z- yönünde hız söz konusu değildir.

$u \neq 0 \;,\; v = w = 0$

4) $g_x = g_z = 0 \;,\; g_y = -g$

5) $\dfrac{\partial P}{\partial x} = \text{sabit}$

Süreklilik Denklemi

$$\frac{\partial u}{\partial x} + \underbrace{\cancel{\frac{\partial v}{\partial y}}}_{\text{Paralel akış}} + \underbrace{\cancel{\frac{\partial w}{\partial z}}}_{\text{Paralel akış}} = 0$$

$$\frac{\partial u}{\partial x} = 0 \qquad (1)$$

Denklem (1) u hızının x-yönünde değişiminin söz konusu olmadığını ifade eder. (Yani akış x-yönünde TAM GELİŞMİŞ 'tir). Bununla birlikte akış daimi ve plaka sonsuz genişlikte olduğu için $(\partial u/\partial z = 0)$ u sadece y'nin fonksiyonudur.

$$u = u(y) \quad ----\quad (2)$$

Navier-Stokes Denklemleri

x-yönündeki

Daimi akış — $v=0$ — $w=0$ — $g_x=0$ — süreklilik — Plakanın genişliği sonsuz

$$\rho\left(\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}}\right) = -\frac{\partial P}{\partial x} + \cancel{\rho g_x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

süreklilikten $\partial u/\partial x=0$

$$0 = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} \quad \text{------} \quad (3)$$

y-yönünde

Paralel akış ($v=w=0$) — Paralel akış ($v=w=0$)

$$\rho\left(\cancel{\frac{\partial v}{\partial t}} + \cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}} + \cancel{w\frac{\partial v}{\partial z}}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}} + \cancel{\frac{\partial^2 v}{\partial z^2}}\right)$$

Daimi akış — $(g_y=-g)$

$$0 = -\frac{\partial P}{\partial y} - \rho g \quad \text{------} \quad (4)$$

z-yönündeki

Daimi — Paralel akış ($v=w=0$) — $g_z=0$ — Paralel akış

$$\rho\left(\cancel{\frac{\partial w}{\partial t}} + \cancel{u\frac{\partial w}{\partial t}} + \cancel{v\frac{\partial w}{\partial y}} + \cancel{w\frac{\partial w}{\partial z}}\right) = -\frac{\partial P}{\partial z} + \cancel{\rho g_z} + \mu\left(\cancel{\frac{\partial^2 w}{\partial x^2}} + \cancel{\frac{\partial^2 w}{\partial y^2}} + \cancel{\frac{\partial^2 w}{\partial z^2}}\right)$$

$$0 = -\frac{\partial P}{\partial z} \quad \text{------} \quad (5)$$

Denklem (5) basıncın z-yönde değişmediğini ifade eder. Denklem (4) aşağıdaki basınç alanı için

$$\frac{\partial P}{\partial y} = -\rho g$$

$$\partial P = -\rho g\, \partial y$$

$$P = -\rho g y + f(x) \quad \text{---------} \quad (6)$$

Denklem (6) basıncın hidrostatik olarak y-yönünde değiştiğini gösterir.

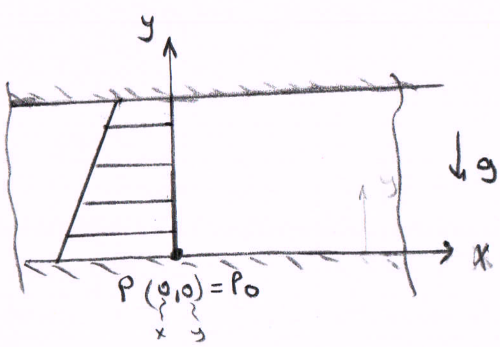
Hatırlatma ✱✱

P'nin hem x'in hem de y'nin fonksiyonu olmasından dolayı Denklem (7)'de integral sabiti yerine $f(x)$ yani x'e bağlı bir fonksiyon eklendiğine dikkat edin. Bunun nedeni y'ye göre kısmi integrasyon işlemi yapmamızdan kaynaklanıyor. Kısmi integrasyon işlemleri yapılırken dikkat edilmelidir.

Denklem (6) basıncın hidrostatik olarak y-yönünde değiştiğini gösterir. Yani Denklem (6) basit bir hidrostatik basınç dağılımını temsil eder. Bu durum bizi hidrostatik basıncın akıştan bağımsız olarak etkidiği sonucuna ulaştırır.

Hatırlatma ✱✱

Serbest yüzeylere sahip olmayan sıkıştırılamaz akış alanları için hidrostatik basınç, akış alanının dinamiğine etkide bulunmaz.



$$P(x,y) = -\rho g y + f(x)$$

$$\frac{\partial P}{\partial x} = c \Rightarrow P(x) = cx + c_1$$

$$P(x,y) = -\rho g y + cx + c_1$$

$$P(x,y) = -\rho g y + \frac{\partial P}{\partial x} x + c_1$$

$$x = 0,\ y = 0 \rightarrow P(x,y) = P_0$$

$$P_0 = 0 + 0 + c_1 \Rightarrow c_1 = P_0$$

$$\boxed{P(x,y) = -\rho g y + \frac{\partial P}{\partial x} x + P_0}$$

Basınç Alanı

Hız alanını belirlemek için Denklem (3)'ü çözelim.

$$\int \frac{d^2u}{dy^2} = \int \frac{1}{\mu}\frac{\partial P}{\partial x}$$

$$\int \frac{du}{dy} = \int \frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right) y + c_1$$

$$u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 + c_1 y + c_2 \quad \text{------} \quad (7)$$

denklemi elde edilir. c_1 ve c_2 integral sabitlerini belirlemek için

(S1) $y = 0 \longrightarrow u = 0$

(S2) $y = h \longrightarrow u = 0$

sınır şartlarını kullanalım:

(S1): $y = 0 \rightarrow u = 0$

$$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) 0^2 + c_1 0 + c_2 \quad \Rightarrow \quad \boxed{c_2 = 0}$$

(S2) $y = h \rightarrow u = 0$

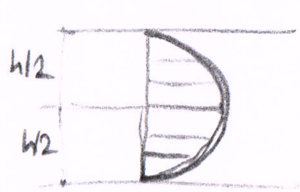
$$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) h^2 + c_1 h \quad \Rightarrow \quad c_1 = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) h$$

Bu durumda hız dağılımı için aşağıdaki ifade elde edilir:

$$u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right) y^2 - \frac{1}{2\mu}\frac{\partial P}{\partial x} h y$$

$$\boxed{u = \frac{1}{2\mu}\frac{\partial P}{\partial x}\left(y^2 - hy\right)} \quad \text{--------} \quad (8)$$

- Akış basınç gradyanı ile doğru orantılı
- Viskozite ile ters orantılı
- Plakalar arasındaki mesafeye bağlı



- Plakalar arasında geçen akışın debisi Q (z-yönündeki birim uzunluk için)

$$Q = \int_0^h u(y)\, dA = \int_0^h u(y)\, dy$$

$$A = y \cdot z = y \cdot (1)$$

$$\frac{dA}{dy} = 1 \Rightarrow \underline{dA = dy}$$

$$= \int_0^h \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(y^2 - hy\right) dy$$

$$= \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)\Bigg|_0^h$$

$$= \frac{1}{2\mu}\frac{\partial P}{\partial x}\left(\frac{h^3}{3} - \frac{h^3}{2}\right) = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\frac{h^3}{6}$$

$$\boxed{Q = -\frac{h^3}{12\mu}\frac{\partial P}{\partial x}} \qquad (9)$$

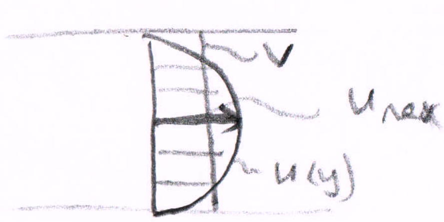
ortalama hız

$$Q = V \cdot A = V \cdot h \cdot (1) = -\frac{h^3}{12\mu}\frac{\partial P}{\partial x}$$

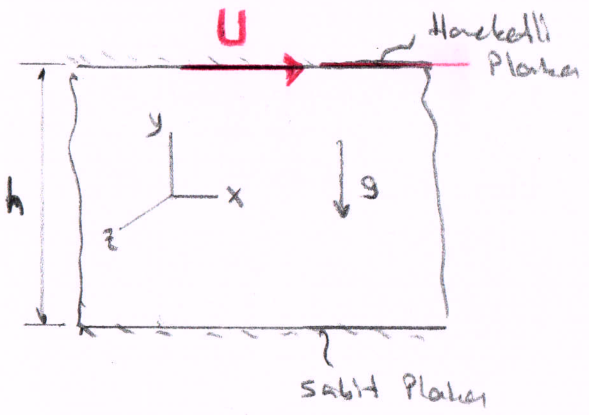
$$\boxed{V = -\frac{h^2}{12\mu}\frac{\partial P}{\partial x}}$$: ortalama hız.

$$\boxed{u_{max} = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left(\frac{h^2}{4} - \frac{h^2}{2}\right) = -\frac{h^2}{8\mu}\frac{\partial P}{\partial x}}$$: maksimum hız

$$\boxed{u_{max} = \frac{3}{2} V}$$



2. Couette Akışı



$$\frac{\partial P}{\partial x} = c = \text{sabit}$$

Hız profili ikeniki plakanın sabit olduğu 1. Problem daha önce elde ettiğimiz denklem (7) ile aynı olacaktır ancak sınır şartlarımız farklı:

(S1) $y=0 \rightarrow u=0$

(S2) $y=h \rightarrow u=U$

sınır şartlarını kullanarak Couette Akışı için hız profilini elde edebilmemiz

(S1) $y=0 \rightarrow u=0$

$$0 = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)0^2 + c_1 0 + c_2 \Rightarrow \boxed{c_2=0}$$

(S2) $y=h \rightarrow u=U$

$$U = \frac{1}{2\mu}\frac{\partial P}{\partial x}h^2 + c_1 h \Rightarrow \boxed{c_1 = \frac{U}{h} - \frac{h}{2\mu}\frac{\partial P}{\partial x}}$$

$$u = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)y^2 + \left(\frac{U}{h} - \frac{h}{2\mu}\frac{\partial P}{\partial x}\right)y$$

$$\boxed{u = U\frac{y}{h} + \frac{1}{2\mu}\frac{\partial P}{\partial x}\left(y^2 - hy\right)}$$ Hız profili

$$\frac{u}{U} = \frac{y}{h} + \frac{1}{2\mu U}\frac{\partial P}{\partial x}\left(y^2 - hy\right)$$ Boyutsuz hız profili

$$\frac{u}{U} = \frac{y}{h} + \frac{1}{2\mu U}\frac{\partial P}{\partial x}\frac{y}{h}\left(\frac{y}{h} + \frac{h}{h}\right)\cdot\left(\frac{h^2}{\cancel{h}}\right)$$

$$\frac{u}{U} = \frac{y}{h} + \frac{h^2}{2\mu U}\frac{\partial P}{\partial x}\frac{y}{h}\left(\frac{y}{h} - 1\right)$$

$$\frac{u}{U} = \frac{y}{h} + \frac{h^2}{2\mu U}\frac{\partial P}{\partial x}\frac{y}{h}\left(\frac{y}{h} - 1\right)$$

$$\boxed{u^* = y^* + \frac{1}{2}P^* y^*\left(y^* - 1\right)}$$

Boyutsuz hız profili