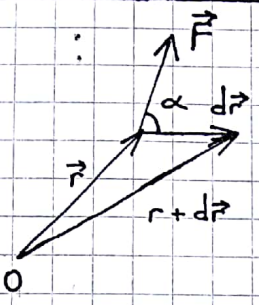
Beer, Johnston, Cornwell
MIM 210
Dinamik

Bölüm 13: Parçacık Kinetiği: Enerji ve Momentum Yöntemleri

13.2. Bir Kuvvetin İşi



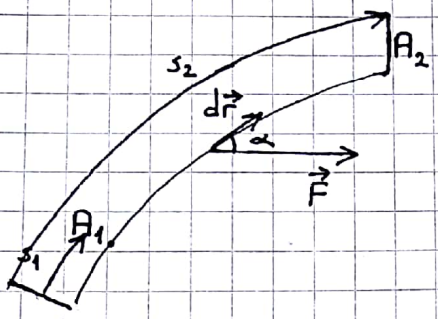
$$dU = \vec{F} \cdot d\vec{r}$$

$$dU = F\,ds\cos\alpha$$

Dik bileşenler cinsinden

$$dU = F_x\,dx + F_y\,dy + F_z\,dz$$

İşin birimi $N \cdot m$ = Joule



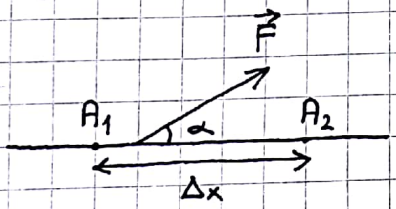
$$U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \cdot d\vec{r}$$

$$U_{1\to 2} = \int_{s_1}^{s_2} F\cos\alpha\,ds = \int_{s_1}^{s_2} F_t\,ds$$

Dik bileşenler cinsinden

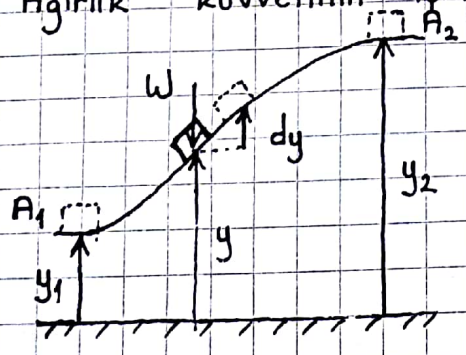
$$U_{1\to 2} = \int_{A_1}^{A_2} (F_x\,dx + F_y\,dy + F_z\,dz)$$

Doğrusal hareketle sabit bir kuvvetin işi:



$$U_{1\to 2} = F\cos\alpha\,\Delta x$$

Ağırlık kuvvetinin işi:



$F_x = 0$, $F_y = -W$ ve $F_z = 0$

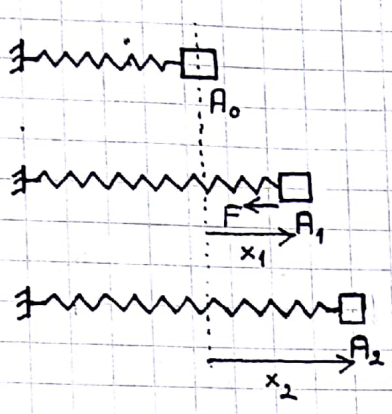
$$dU = -W\,dy$$

$$U_{1\to 2} = -\int_{y_1}^{y_2} W\,dy = W_{y1} - W_{y2}$$

$$U_{1\to 2} = -W(y_2 - y_1) = -W\,\Delta y$$

$\Delta y < 0$ iken iş pozitif olur.

Yay kuvvetinin işi:



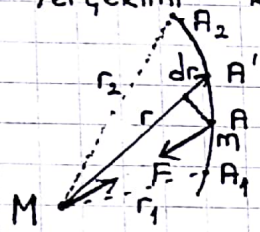
$F = kx$

Cisim A_1'de A_2'ye giderken

$$dU = -F\,dx = -kx\,dx$$

$$U_{1\to 2} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

Yerçekimi kuvvetinin işi:

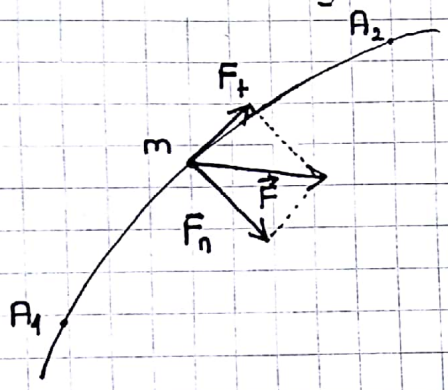


$$F = G.\frac{Mm}{r^2}$$

$$dU = -F\,dr = -G.\frac{Mm}{r^2}\,dr$$

$$U_{1\to 2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

13.3. Bir Parçacığın Kinetik Enerjisi : İş ve Enerji İlkesi



$$F_t = ma_t \quad \text{veya} \quad F_t = m\frac{dv}{dt}$$

$$F_t = m\frac{dv}{ds}\cdot\frac{ds}{dt} = mv.\frac{dv}{ds}$$

$$F_t\,ds = mv\,dv$$

Cisim A_1'den A_2'ye giderken

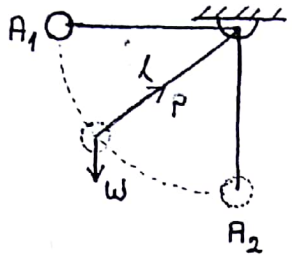
$$\int_{s_1}^{s_2} F_t\,ds = m\int_{v_1}^{v_2} v\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$T = \frac{1}{2}mv^2$ kinetik enerji $\Rightarrow$ $U_{1\to 2} = T_2 - T_1 = \Delta T$

İş-kinetik enerji teoremi

Birim $kg.\left(\frac{m}{s}\right)^2 = (kg.m/s^2).m = N.m = \text{Joule}$

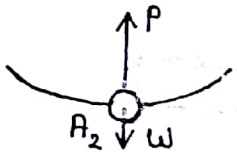
13.4. İş ve Enerji İlkesinin Uygulamaları



$$T_1 + U_{1\to 2} = T_2$$

$$0 + Wl = \frac{1}{2}\frac{W}{g}v_2^2$$

$$v_2 = \sqrt{2gl}$$



$$\Sigma F_t = ma_t = 0$$

$$\Sigma F_n = ma_n$$

$$P - W = ma_n = \frac{W}{g}\cdot\frac{v_2^2}{l} = \frac{W}{g}\cdot\frac{2gl}{l} = 2W$$

$$P = 3W$$

13.5. Güç ve Verim

Ortalama güç $= \frac{\Delta U}{\Delta t}$

$\Delta t \to 0$ durumda Güç $= \frac{dU}{dt}$

$$\text{Güç} = \frac{dU}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt}$$

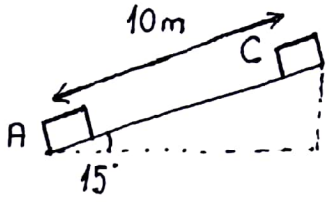
$\vec{F}$ sabitse Güç $= \vec{F}\cdot\vec{v}$

Gücün birimi $1\,W = 1\,J/s = 1\,N\cdot m/s$

Mekanik verim η

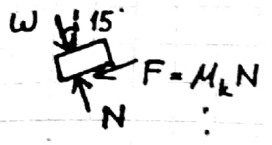
$$\eta = \frac{\text{çıkan iş}}{\text{giren iş}} = \frac{\text{güç çıkışı}}{\text{güç girişi}} < 1$$

Örnek:



Cisim A noktasından v_A hızıyla fırlatılınca C noktasına sıfır hızla ancak erişiyor. Cisim ve eğik düzlem arasındaki sürtünme katsayısı 0,12 ise

a. $v_A = ?$ b. cisim A noktasına geri döndüğünde $v_A = ?$



a. Yukarı doğru hareket $v_c = 0$

$T_A = \frac{1}{2} m v_A^2$, $T_c = 0$

$U_{A \to C} = (-W \sin 15^\circ - F) \cdot 10\,m$

$\Sigma F = N - W \cos 15^\circ = 0 \quad \Rightarrow N = W \cos 15^\circ$

$F = \mu_k N = 0{,}12\, W \cos 15^\circ$

$U_{A \to C} = -W (\sin 15^\circ + 0{,}12 \cos 15) \cdot 10\,m$

$T_A + U_{A \to C} = T_c = 0 \qquad \frac{1}{2} \frac{W}{g} v_A^2 - W(\sin 15^\circ + 0{,}12 \cos 15^\circ) \cdot 10\,m = 0$

$v_A^2 = 2 \cdot 9{,}81 \cdot (\sin 15^\circ + 0{,}12 \cos 15^\circ)\, 10\,m$

$v_A^2 = 73{,}5 \qquad v_A = 8{,}57\ m/s$

b. Aşağı doğru hareket C'den A'ya doğru

$T_c = 0 \qquad T_A = \frac{1}{2} m v_A^2 \qquad U_{C \to A} = (W \sin 15^\circ - F) \cdot 10$

$T_c + U_{C \to A} = T_A \qquad 0 + W \cdot (\sin 15^\circ - 0{,}12 \cos 15^\circ)\, 10 = \frac{1}{2} m v_A^2 = \frac{1}{2} \frac{W}{g} v_A^2$

$v_A^2 = 2 \cdot 9{,}81 \cdot (\sin 15^\circ - 0{,}12 \cos 15^\circ)\, 10\,m$

$v_A^2 = 28{,}039$

$v_A = 5{,}3\ m/s$