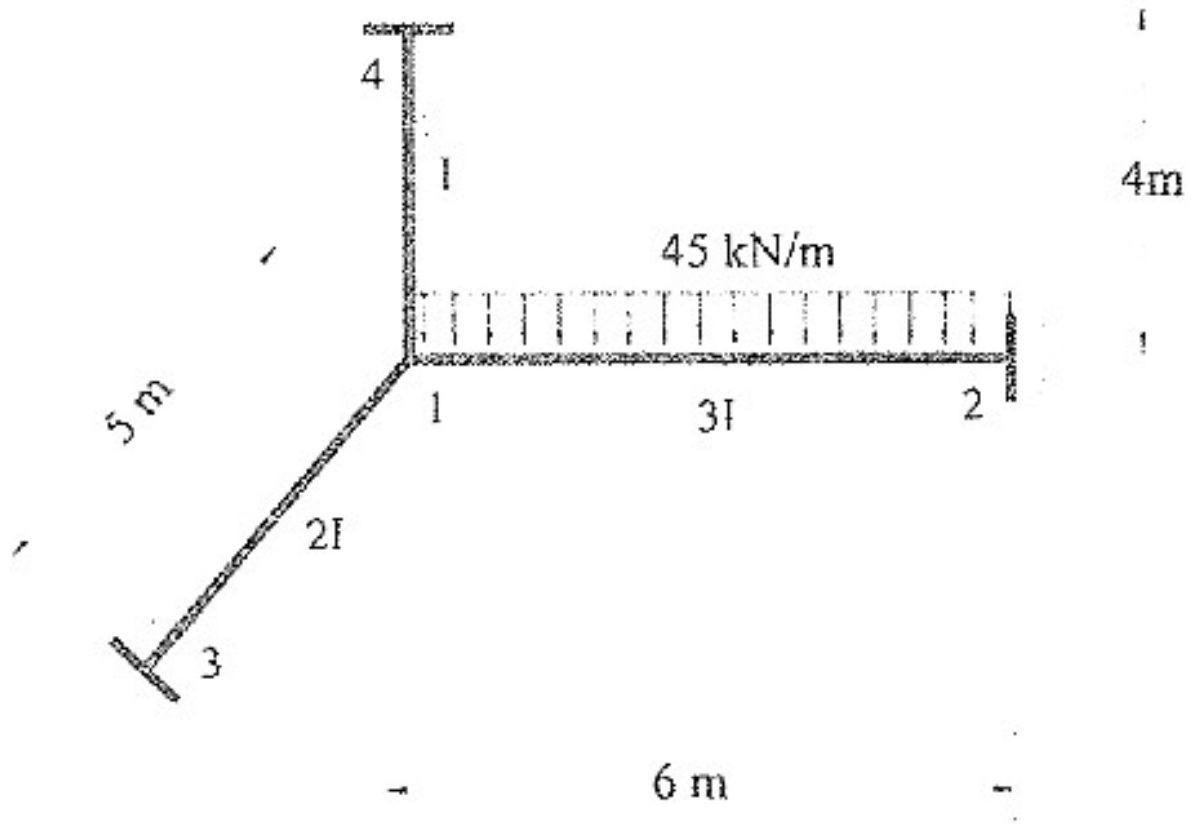
YAPI STATİĞİ II

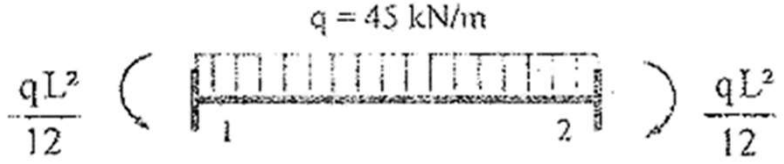
KUVVET METODU
AÇI METODU

Hazırlayan: Yard.Doç.Dr.Kıvanç TAŞKIN

Örnek 3:



ii. *Ankastrelik uç kuvvetleri*



$$M_{12} = \frac{qL^2}{12} = \frac{45\times 6^2}{12} = 135 \text{ kNm}, \qquad M_{12} = -\frac{qL^2}{12} = -\frac{45\times 6^2}{12} = -135 \text{ kNm}$$

iii. *Birim yerdeğiştirme sabitleri*

Çubuk i-j	$m_{i\theta_i}$	$m_{i\theta_j}$
1-3	4E(2I) / 5 **1.60EI**	2E(2I) / 5 **0.80EI**
1-4	4E(I) / 4 **1.00EI**	2E(I) / 4 **0.50EI**
1-2	4E(3I) / 6 **2.00EI**	2E(3I) / 6 **1.00EI**

iv. *Denge denklemleri*

$$\sum M_1 = 0 \quad \Rightarrow \quad M_{13}+M_{12}+M_{14}=0$$

$$M_{13} = m_{1\theta_1}^{13} \times \theta_1 \ , \ M_{14} = m_{1\theta_1}^{14} \times \theta_1 \ , \ M_{12} = M_{12}^{*} + m_{1\theta_1}^{12} \times \theta_1$$

$$(m_{1\theta_1}^{13} + m_{1\theta_1}^{14} + m_{1\theta_1}^{12}) \times \theta_1 + M_{12}^{*} = 0$$

$$(1.60EI + 1.00EI + 2.00EI) \times \theta_1 + 135 = 0 \quad \Rightarrow \quad \theta_1 = -29.35/EI \quad [\text{rad}]$$

v. *Düğüm noktalarına komşu çubuk kesitlerindeki uç momentleri*

$$M_{13} = m_{1\theta_1}^{13} \times \theta_1 = 1.6\ \text{EI} \times (-29.35/EI) = -46.96 \quad \text{kNm}$$

$$M_{14} = m_{1\theta_1}^{14} \times \theta_1 = 1.0\ \text{EI} \times (-29.35/EI) = -29.35 \quad \text{kNm}$$

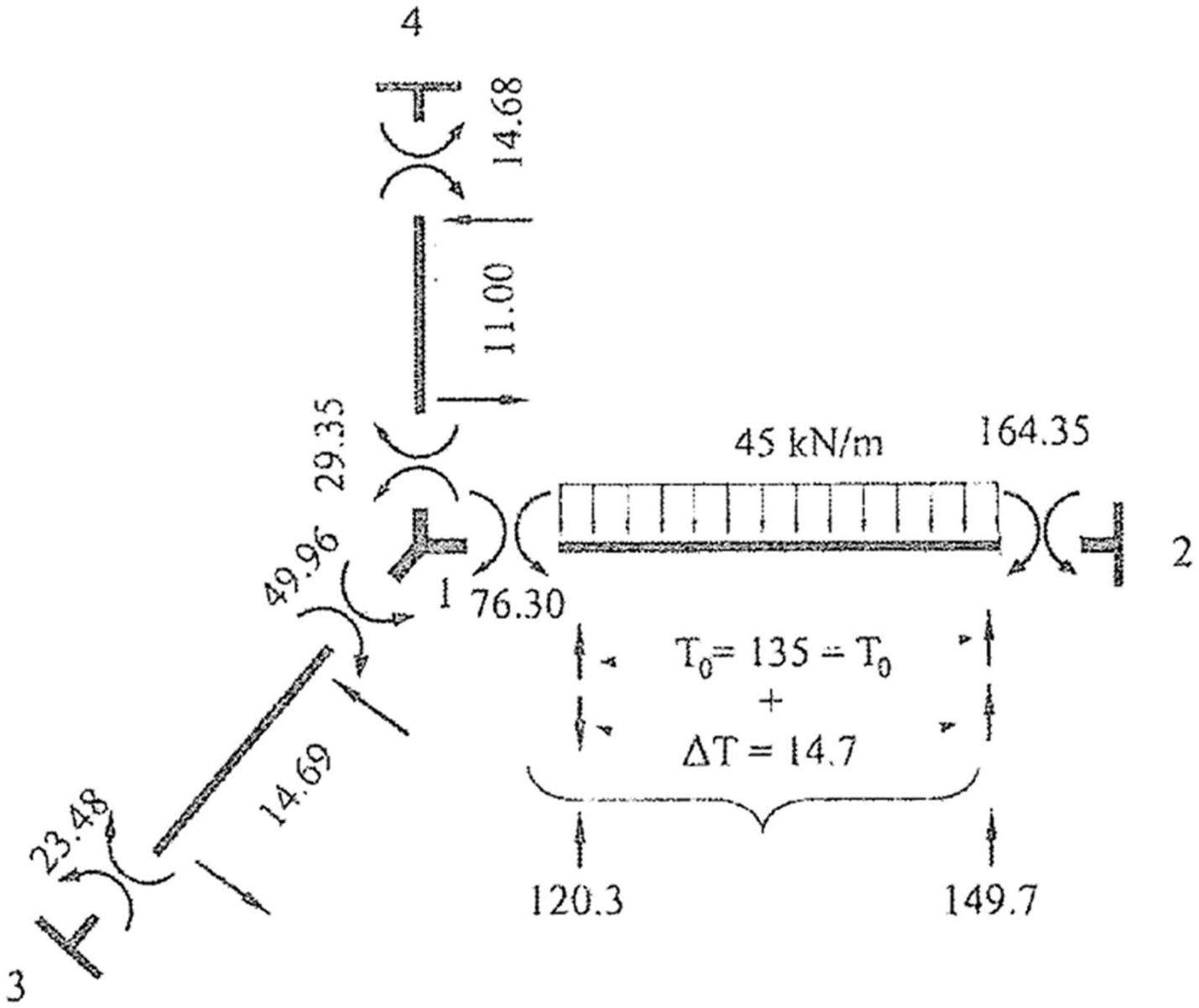
$$M_{12} = M_{12}^{*} + m_{1\theta_1}^{12} \times \theta_1 = 135 + 2.0\ \text{EI} \times (-29.35/EI) = 76.30 \quad \text{kNm}$$

vi. *Karşı uçlardaki momentler – Mesnet momentleri*

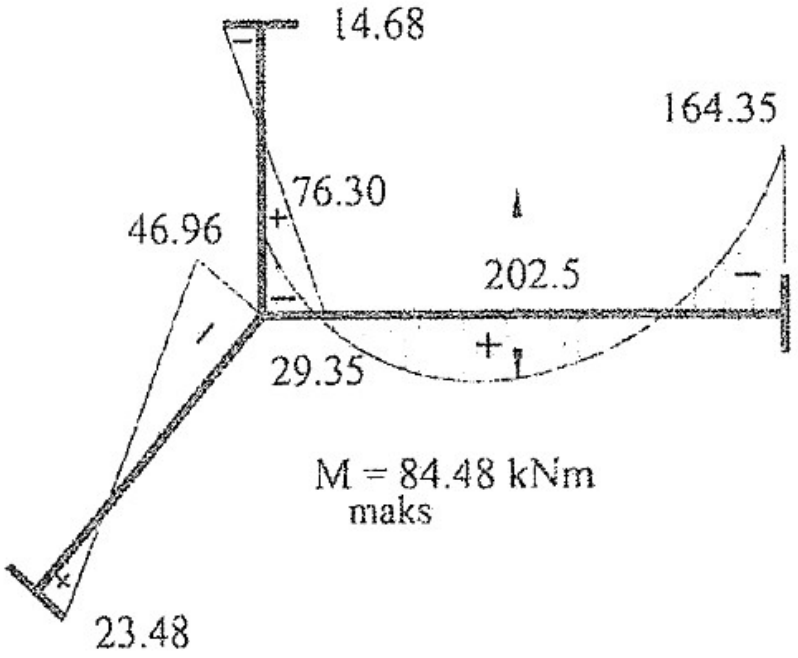
$$M_{31} = m_{3\theta_1}^{31} \times \theta_1 = 0.8\ \text{EI} \times (-29.35/EI) = -23.48\ \text{kNm}$$

$$M_{41} = m_{4\theta_1}^{41} \times \theta_1 = 0.5\ \text{EI} \times (-29.35/EI) = -14.68\ \text{kNm}$$

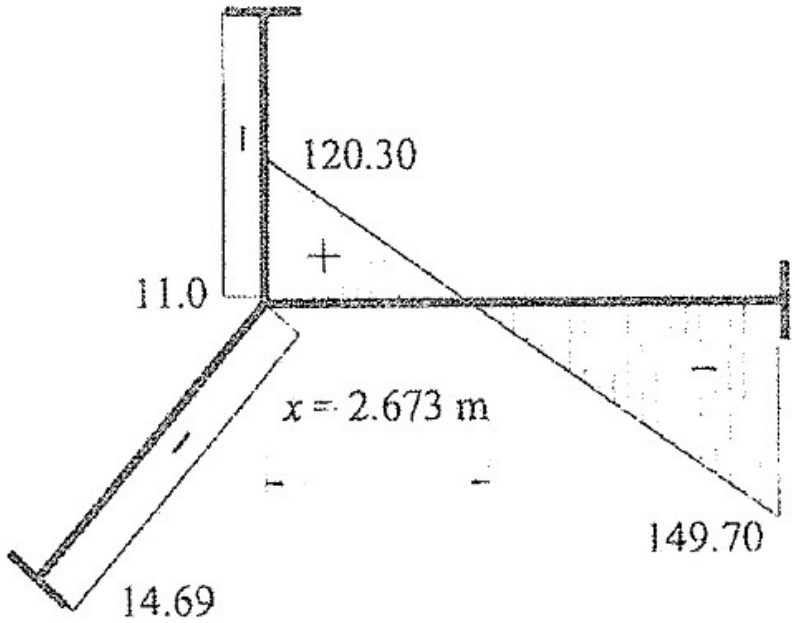
$$M_{21} = M_{12} + m_{2\theta_1}^{21} \times \theta_1 = -135 + 1.0\ \text{EI} \times (-29.35/EI) = -164.35\ \text{kNm}$$


4
14.68
11.00
29.35
45 kN/m
164.35
49.96
1
76.30
2
$T_0 = 135 = T_0$
+
$\Delta T = 14.7$
14.69
23.48
3
120.3
149.7

viii. M eğilme momenti ve T kesme kuvveti diyagramları

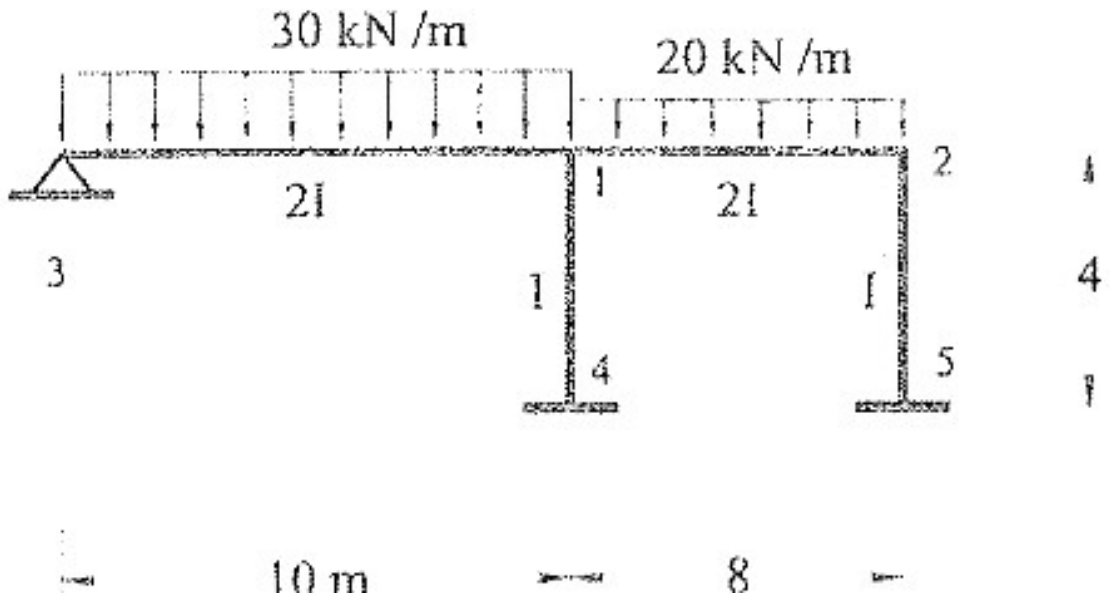


M [kNm]



T [kN]

Örnek 4:



Yapısal özellikleri ile dış yükleri verilen sistem yerdeğiştirme -*açı*- yöntemi ile çözümlenecektir. Doğru eksenli çubuklarda eksenel boy değişimleri yok sayılarak, $\Delta=0$ alınacaktır.

i. Bilinmeyenler ve denge denklemleri

Sistemin *4* ve *5* noktaları ankastre, *3* noktası mafsallı olmak üzere mesnetleri, *1* ve *2* noktaları ise çubukların birbirine bağlandığı düğüm noktalarıdır. Doğru eksenli çubuklarda $\Delta=0$ öngörüldüğü için sistemin düğüm noktalarında ötelenme yoktur, sadece düzleme dik eksen etrafında dönme mevcuttur. Buna göre;

Bilinmeyenler : θ_1, θ_2

Denklemler : $\sum M_1 = 0$, $\sum M_2 = 0$ olacaktır.

ii. *Ankastrelik uç kuvvetleri*

$$M_{13} = -\frac{qL^2}{8} = -\frac{30\times 10^2}{8} = -375 \text{ kNm}$$

$$M_{12} = \frac{qL^2}{12} = \frac{20\times 8^2}{12} = 106.67 \text{ kNm}, \qquad M_{12} = -\frac{qL^2}{12} = -\frac{20\times 8^2}{12} = -106.67 \text{ kNm}$$

Birim yerdeğiştirme sabitleri

Çubuk i-j	$m_{i\theta_i}$	$m_{i\theta_j}$
1-3	3E(2I) / 10 **0.60EI**	-
1-4	4E(I) / 4 **1.00EI**	2E(I) / 4 **0.50EI**
1-2	4E(2I) / 8 **1.00EI**	2E(2I) / 8 **0.50EI**
2-5	4E(I) / 4 **1.00EI**	2E(I) / 4 **0.50EI**

iii. *Denge denklemleri*

I düğüm noktası moment denge denklemi;

$$\sum M_1 = 0 \quad \Rightarrow \quad M_{13} + M_{12} + M_{14} = 0$$

$$M_{13} = \overline{M_{13}} + m_{1\theta_1}^{13} \times \theta_1 \qquad M_{14} = m_{1\theta_1}^{14} \times \theta_1 \qquad M_{12} = \overline{M_{12}} + m_{1\theta_1}^{12} \times \theta_1 + m_{1\theta_2}^{12} \times \theta_2$$

$$M_{13} + M_{12} + M_{14} = \overline{M_{13}} + \overline{M_{12}} + \left(m_{1\theta_1}^{13} + m_{1\theta_1}^{12} + m_{1\theta_1}^{14} \right) \times \theta_1 + m_{1\theta_2}^{12} \times \theta_2 = 0$$

Sayısal değerler yukarıdaki denklemde yerine konur ve EI'lar bir kenara bırakılırsa,

$$-375 + 106.67 + (0.6+1.0+1.0)\ \theta_1 + 0.5\,\theta_2 = 0$$

2 düğüm noktası moment denge denklemi;

$$\sum M_2 = 0 \quad \Rightarrow \quad M_{21} + M_{25} = 0$$

$$M_{25} = m_{2\theta_2}^{25} \times \theta_2 \qquad M_{21} = M_{21}^{*} + m_{2\theta_1}^{12} \times \theta_1 + m_{2\theta_2}^{12} \times \theta_2$$

Sayısal değerler yukarıdaki denklemde yerine konur ve EI'lar bir kenara bırakılırsa,

$$-106.67 + 0.5\,\theta_1 + (1.0+1.0)\,\theta_2 = 0$$

$$\begin{bmatrix} 2.60 & 0.50 \\ 0.50 & 2.00 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 268.33 \\ 106.67 \end{bmatrix} \longrightarrow \begin{aligned} \theta_1 &= 97.641 / EI\ [rad] \\ \theta_2 &= 28.925 / EI\ [rad] \end{aligned}$$

iv. *Uç momentleri*

$$M_{13} = \overline{M_{13}} + \overline{m}^{13}_{1\theta_1} \times \theta_1 = -375 + 0.6 \times 97.64 = -316.42$$

$$M_{14} = m^{14}_{1\theta_1} \times \theta_1 = 1 \times 97.64 = 97.64$$

$$M_{41} = m^{14}_{4\theta_1} \times \theta_1 = 0.5 \times 97.64 = 48.82$$

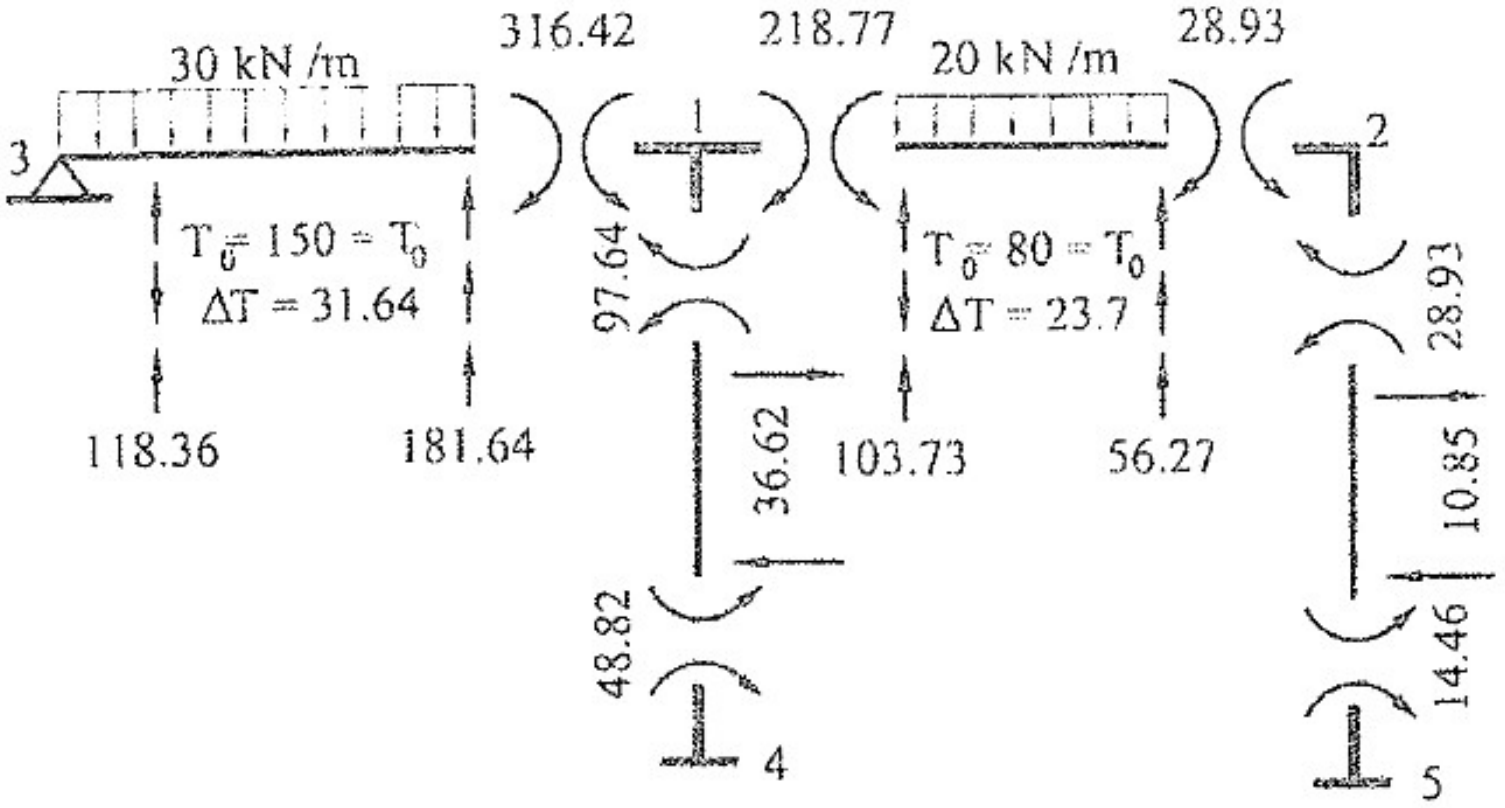
$$M_{12} = M_{12} + m^{12}_{1\theta_1} \times \theta_1 + m^{12}_{1\theta_2} \times \theta_2 = 106.67 + 1 \times 96.64 + 0.5 \times 28.925 = 218.77$$

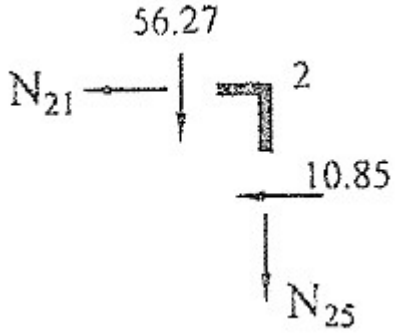
$$M_{21} = M_{21} + m^{12}_{2\theta_1} \times \theta_1 + m^{12}_{2\theta_2} \times \theta_2 = -106.67 + 0.5 \times 96.64 + 1 \times 28.925 = -28.925$$

$$M_{25} = m^{25}_{2\theta_2} \times \theta_2 = 1 \times 28.925 = 28.93$$

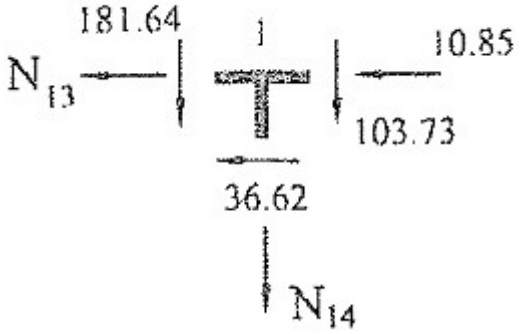
$$M_{52} = m^{25}_{5\theta_2} \times \theta_2 = 0.5 \times 28.925 = 14.46$$

v. *Dönüştürme çizemi; çubuk denge denklemleri ve düğüm noktası izdüşüm denge denklemleri*



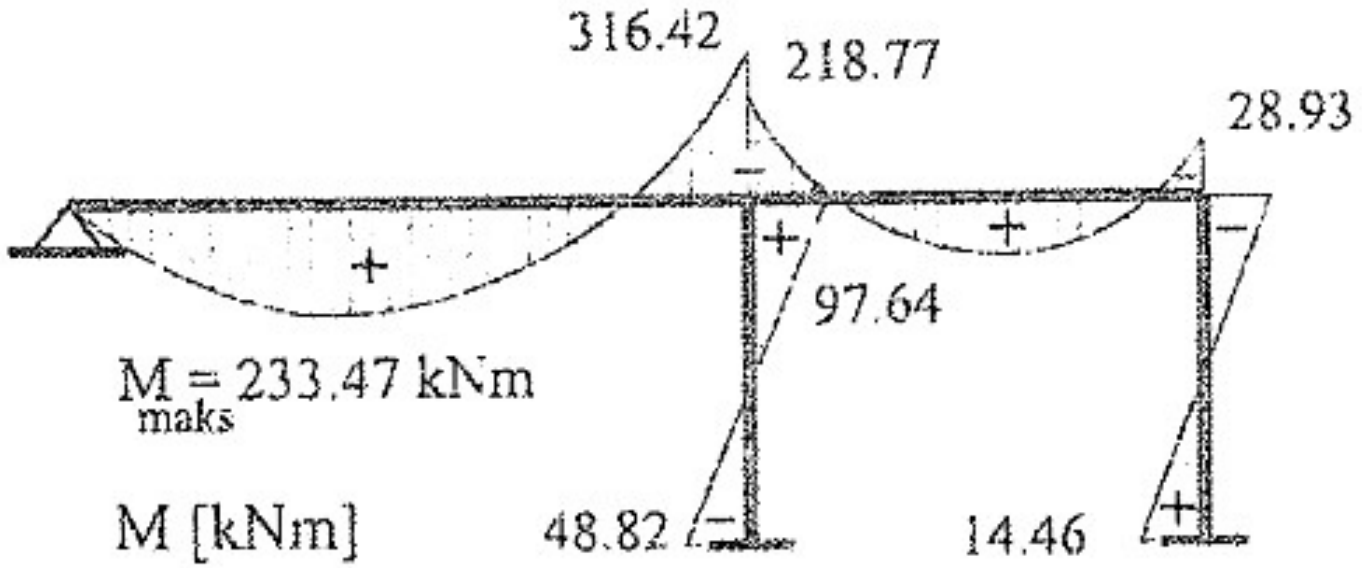


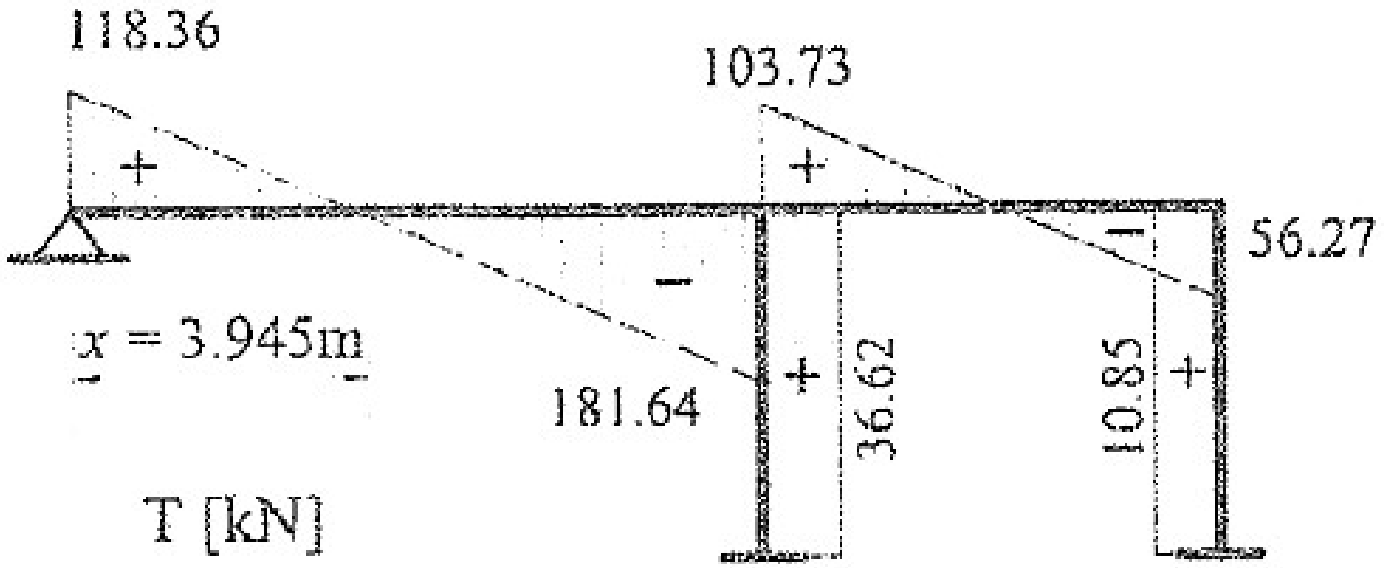
$$\sum X = 0 \Rightarrow N_{21} = -10.85 \text{ kN}$$
$$\sum Y = 0 \Rightarrow N_{25} = -56.27 \text{ kN}$$

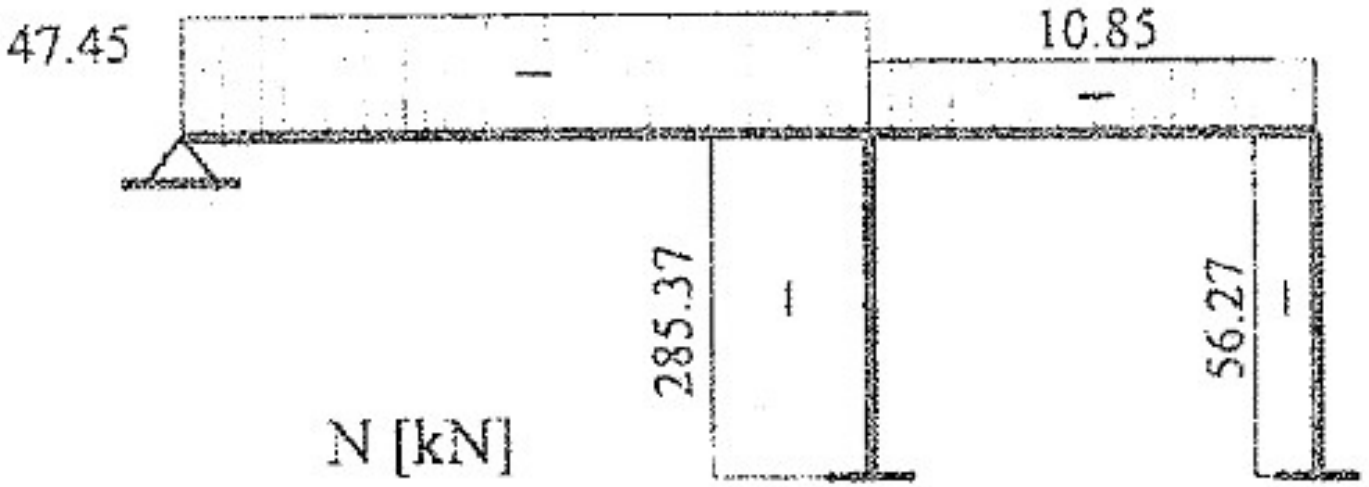


$$\sum X = 0 \Rightarrow N_{13} = -36.62 - 10.85 = -47.45 \text{ kN}$$
$$\sum Y = 0 \Rightarrow N_{14} = -103.73 - 181.64 = -285.37 \text{ kN}$$

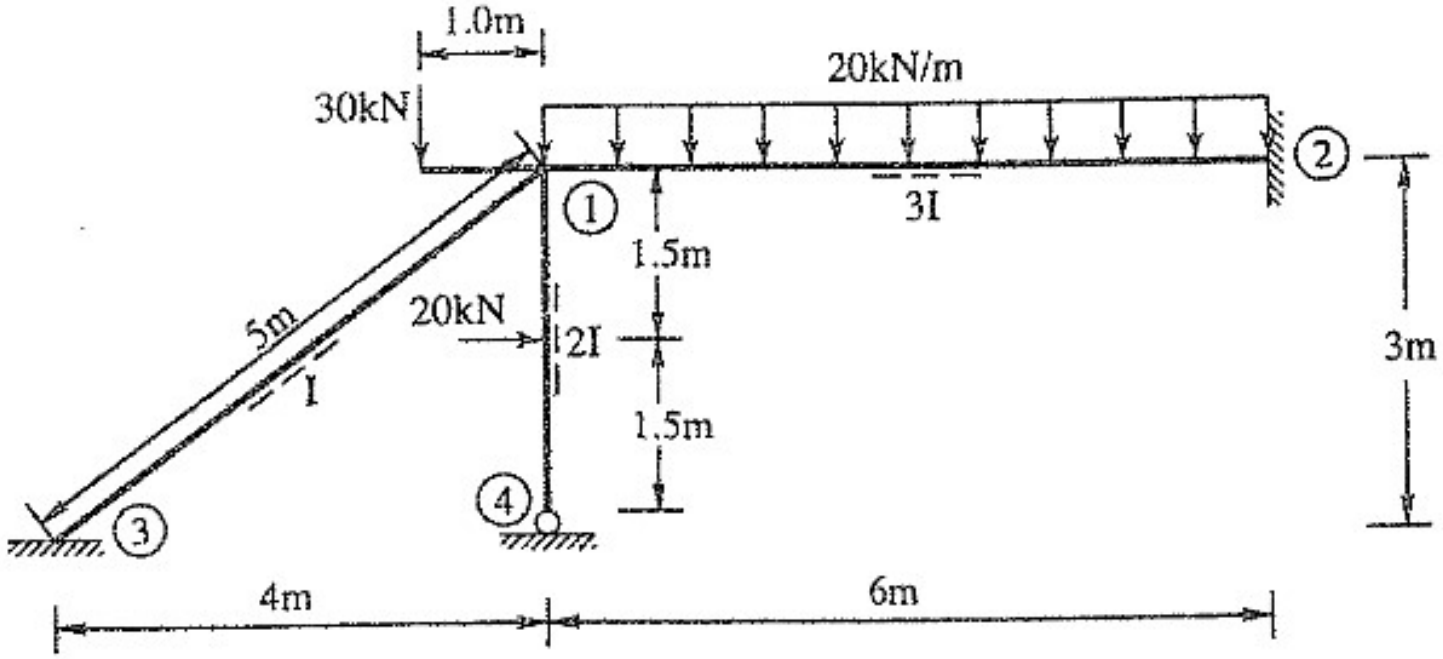
vi. *M eğilme momenti , T kesme kuvveti ve N eksenel kuvvet diyagramları*




118.36
103.73
56.27
+
+
−
x = 3.945m
181.64
+
36.62
10.85
−
+
T [kN]


47.45
10.85
285.37
56.27
N [kN]

Örnek 5:



Şekildeki sistemin açı metodu ile M ve T diyagramlarını çiziniz.

Soruda verilen sistem düğüm noktası sabit sistemdir. (dnss)

1) Bilinmeyen: θ_1

2) Birim Deplasman Sabitleri (bds): Hesapların kolay yapılabilmesi amacıyla $EI = 1$ olarak dikkate alınacaktır.

(31) çubuğu: $m_{i\theta_i} = \frac{4EI}{L} = \frac{4\times1}{5} = 0.80$, $m_{i\theta_j} = \frac{2EI}{L} = \frac{2\times1}{5} = 0.40$

(14) çubuğu: $\overline{m}_{i\theta_i} = \frac{3EI}{L} = \frac{3\times2}{3} = 2.00$

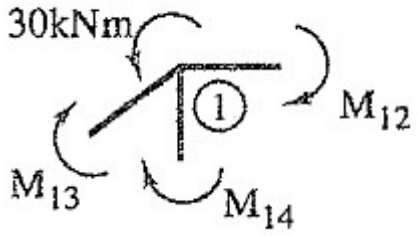
(12) çubuğu: $m_{i\theta_i} = \frac{4\times3}{6} = 2.00$, , $m_{i\theta_j} = \frac{2\times3}{6} = 1.00$

3) Ankastrelik Momentleri:

(14) çubuğu: $\overline{M}_{14} = -\frac{3}{16}PL = -\frac{3}{16}\times20\times3 = -11.25$ kNm

(12) çubuğu: $M_{12} = -M_{21} = \frac{qL^2}{12} = \frac{20\times36}{12} = 60$ kNm

4) Düğüm noktalarında moment denge denklemlerinin yazılması:



$$(\circlearrowleft +)\ \sum M_1 = 0 \Rightarrow M_{12} + M_{14} + M_{13} - 30 = 0$$

$$\Rightarrow \theta_1 \overset{12}{m}_{1\theta_1} + \mathcal{M}_{12} + \theta_1 \overset{14}{m}_{1\theta_1} + \overline{\mathcal{M}}_{14} + \theta_1 \overset{13}{m}_{1\theta_1} - 30 = 0$$

$$\Rightarrow \theta_1 (2.00 + 2.00 + 0.80) + 60 - 11.25 - 30 = 0$$

$$\Rightarrow \theta_1 = -3.91 \left(-\frac{3.91}{EI}\right)$$

5) Uç Momentlerinin bulunması ve sol uçtaki moment düzeltmelerinin yapılması:

$$M_{31} = \theta_1 \overset{13}{m}_{3\theta_1} = -3.91 \times 0.40 = -1.56 \ (1.56)$$

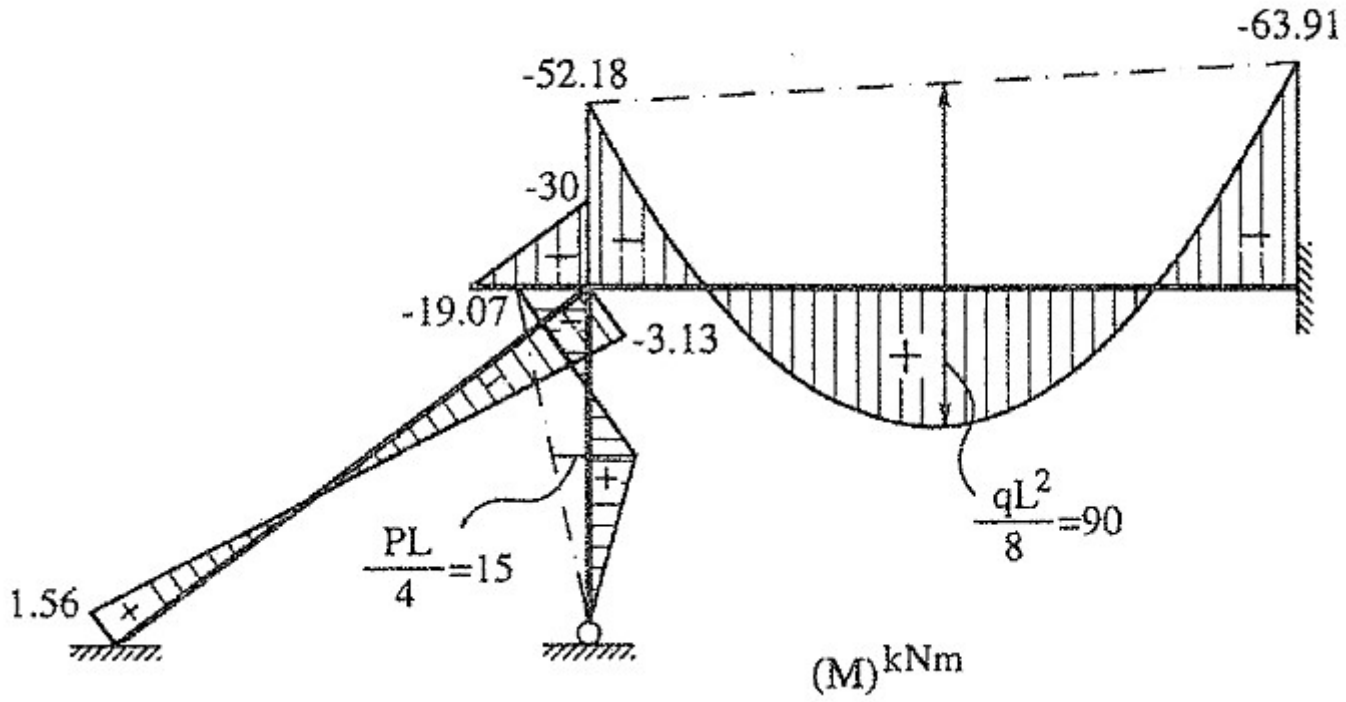
$$M_{13} = \theta_1 \overset{13}{m}_{1\theta_1} = -3.91 \times 0.80 = -3.13$$

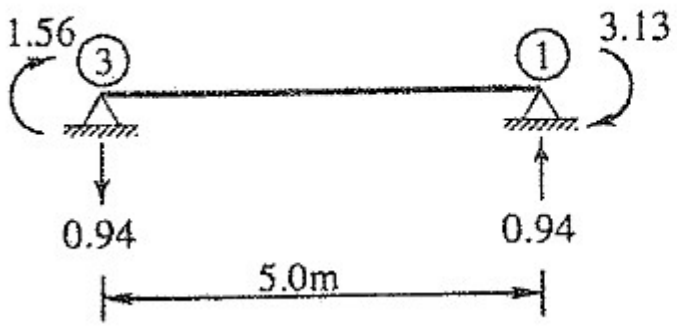
$$M_{14} = \theta_1 \overset{14}{\underline{m}}_{1\theta_1} + \overline{\mathcal{M}_{14}} = -3.91 \times 2.00 - 11.25 = -19.07$$

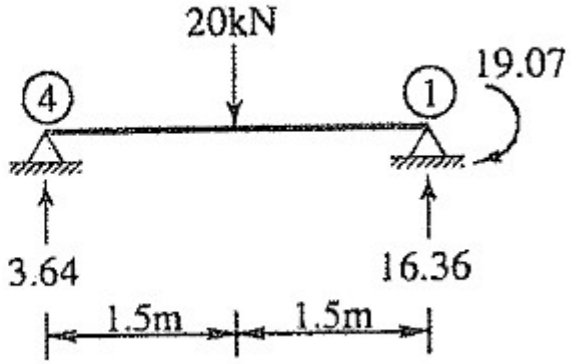
$$M_{12} = \theta_1 \overset{12}{m}_{1\theta_1} + \mathcal{M}_{12} = -3.91 \times 2.00 + 60 = 52.18 \ (-52.18)$$

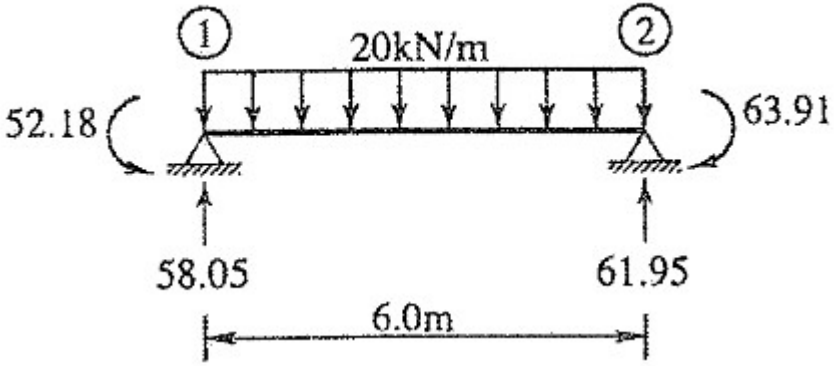
$$M_{21} = \theta_1 \overset{12}{m}_{2\theta_1} + \mathcal{M}_{21} = -3.91 \times 1.00 - 60 = -63.91$$

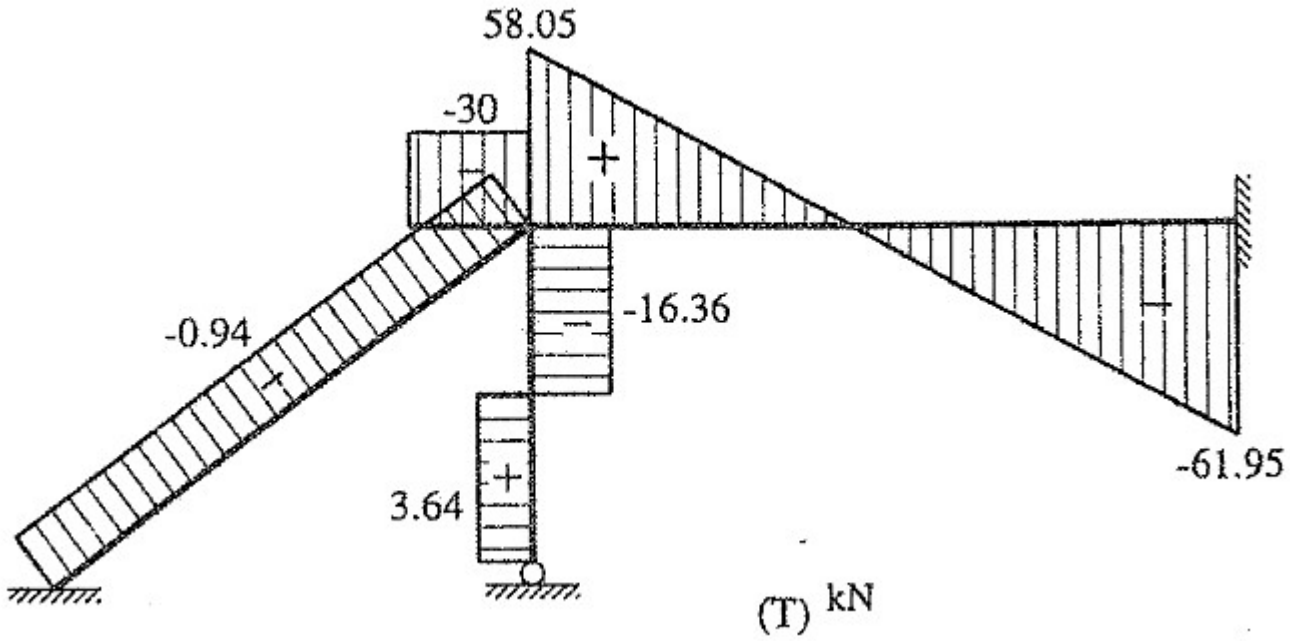
6) M ve T diyagramlarının çizimi:



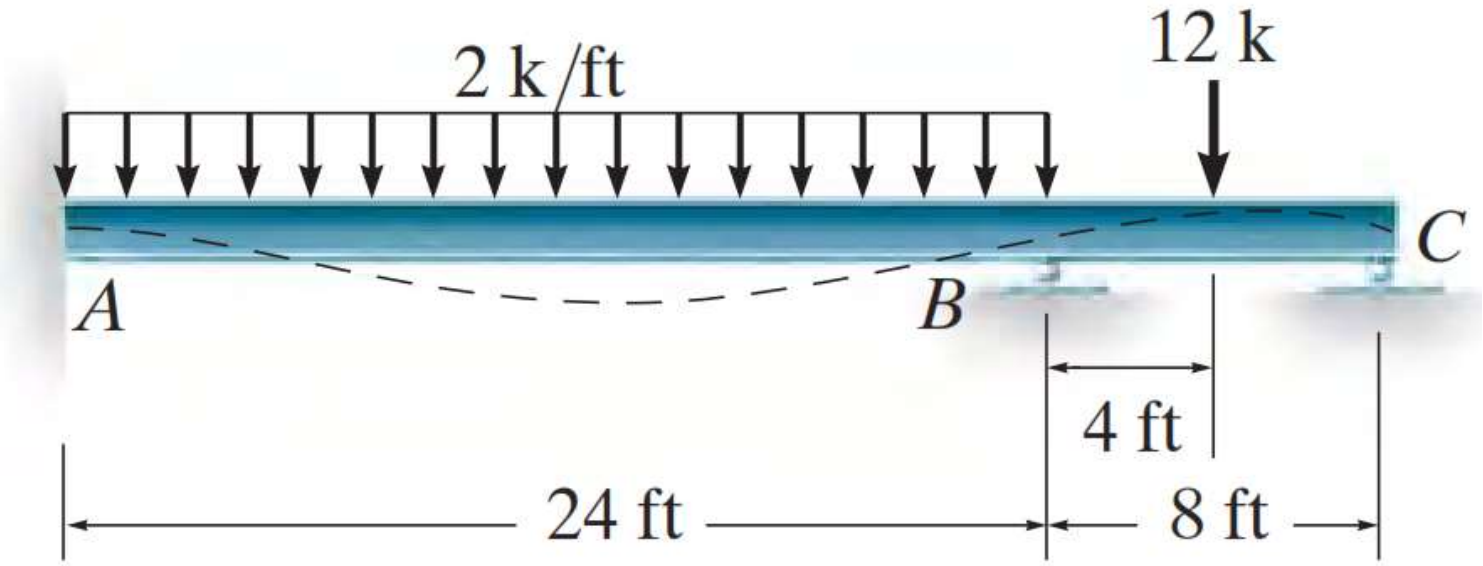

1.56
3
1
3.13
0.94
0.94
5.0m


20kN
4
1
19.07
3.64
16.36
1.5m
1.5m


1
20kN/m
2
52.18
63.91
58.05
61.95
6.0m

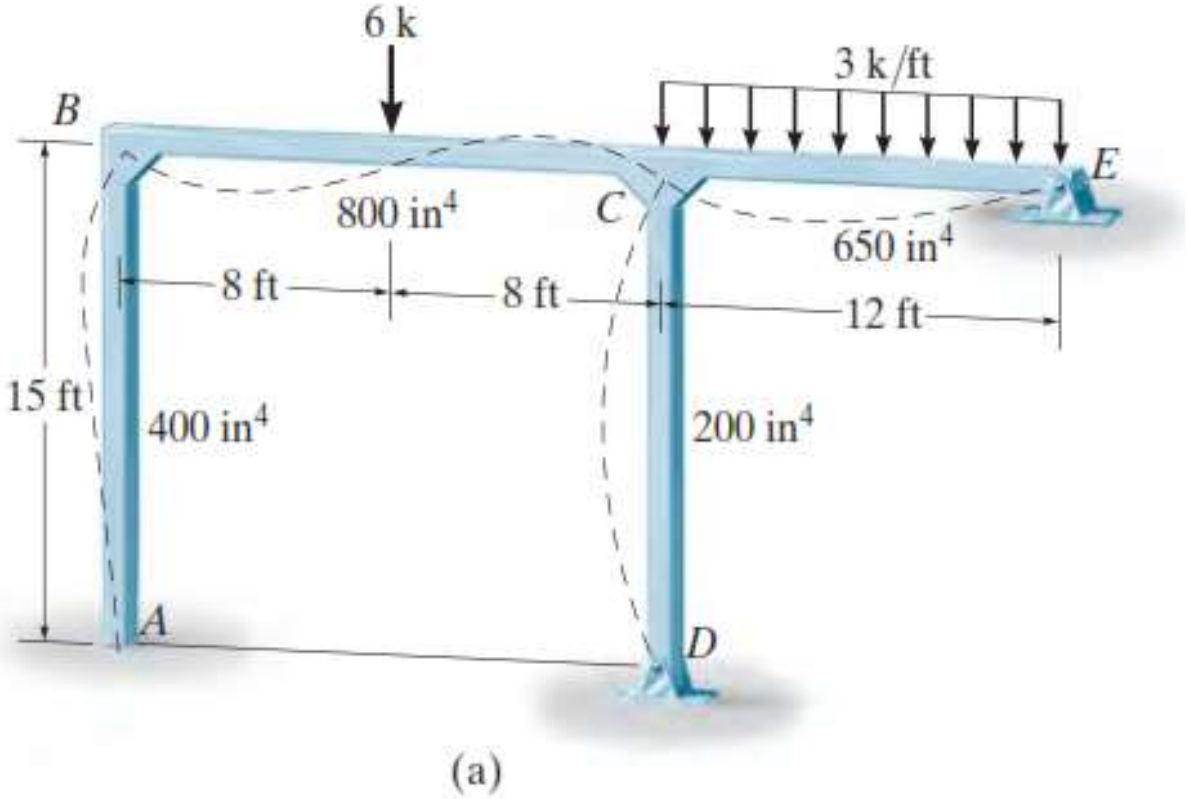

58.05
-30
-0.94
-16.36
3.64
-61.95
(T) kN

ÖDEV 5:



Şekildeki sistemin açı metodu ile M ve T diyagramlarını çiziniz. (EI sabit)

Örnek 6:



Açı yöntemini kullanarak şekildeki sistemin her bir düğüm noktasındaki moment değerlerini bulunuz. (EI= 29 .10^3)

Ankastrelik Momentleri:

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{PL}{8} = -\frac{6(16)}{8} = -12\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{PL}{8} = \frac{6(16)}{8} = 12\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CE} = -\frac{wL^2}{8} = -\frac{3(12)^2}{8} = -54\ \text{k}\cdot\text{ft}$$

$$\theta_A = 0, \qquad \psi_{AB} = \psi_{BC} = \psi_{CD} = \psi_{CE} = 0$$

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2[29(10^3)(12)^2](0.001286)[2(0) + \theta_B - 3(0)] + 0 = 10740.7\theta_B$$

$$M_{BA} = 2[29(10^3)(12)^2](0.001286)[2\theta_B + 0 - 3(0)] + 0 = 21\,481.5\theta_B$$

$$M_{BC} = 2[29(10^3)(12)^2](0.002411)[2\theta_B + \theta_C - 3(0)] - 12$$

$$= 40\,277.8\theta_B + 20\,138.9\theta_C - 12$$

$$M_{CB} = 2[29(10^3)(12)^2](0.002411)[2\theta_C + \theta_B - 3(0)] + 12$$

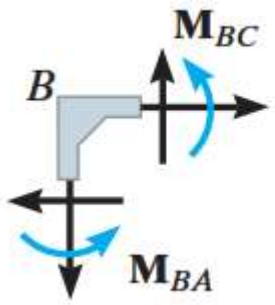
$$= 20\,138.9\theta_B + 40\,277.8\theta_C + 12$$

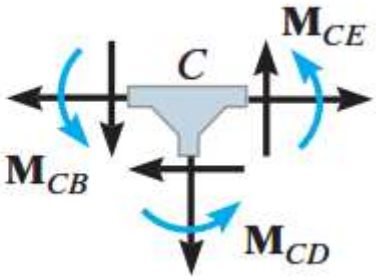
$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = 3[29(10^3)(12)^2](0.000643)[\theta_C - 0] + 0 = 8055.6\theta_C$$

$$M_{CE} = 3[29(10^3)(12)^2](0.002612)[\theta_C - 0] - 54 = 32\,725.7\theta_C - 54$$

Düğüm noktalarında moment denge denklemlerinin yazılması:





$$M_{BA} + M_{BC} = 0$$

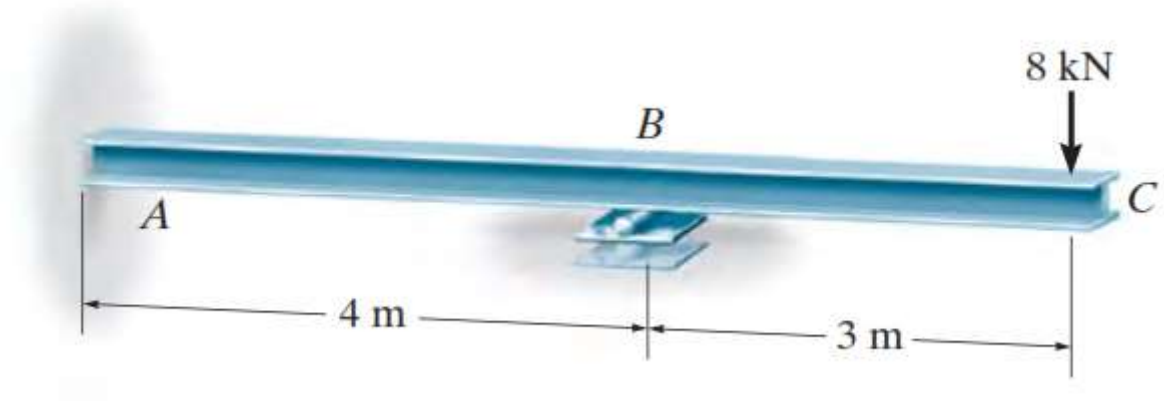
$$M_{CB} + M_{CD} + M_{CE} = 0$$

$$61\,759.3\theta_B + 20\,138.9\theta_C = 12$$
$$20\,138.9\theta_B + 81\,059.0\theta_C = 42$$

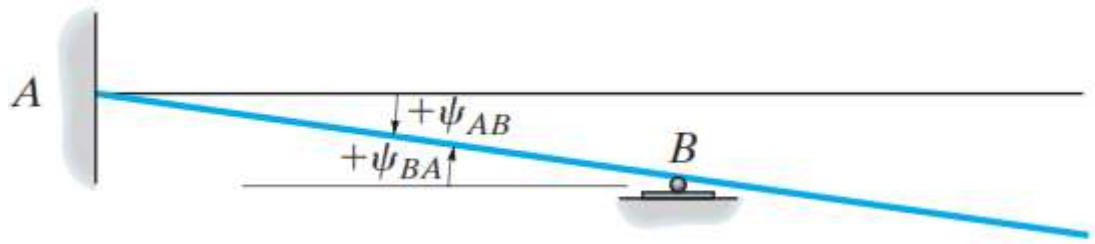
$$\theta_B = 2.758(10^{-5})\ \text{rad} \qquad \theta_C = 5.113(10^{-4})\ \text{rad}$$

$$M_{AB} = 0.296\ \text{k} \cdot \text{ft}$$
$$M_{BA} = 0.592\ \text{k} \cdot \text{ft}$$
$$M_{BC} = -0.592\ \text{k} \cdot \text{ft}$$
$$M_{CB} = 33.1\ \text{k} \cdot \text{ft}$$
$$M_{CD} = 4.12\ \text{k} \cdot \text{ft}$$
$$M_{CE} = -37.3\ \text{k} \cdot \text{ft}$$

Örnek 7:



B noktası 80 mm oturma yapıyorsa , şekilde gösterilmekte olan kirişin A ve B noktalarındaki momentini hesaplayınız. I = $5(10^6)mm^4$ E = 200 Gpa.



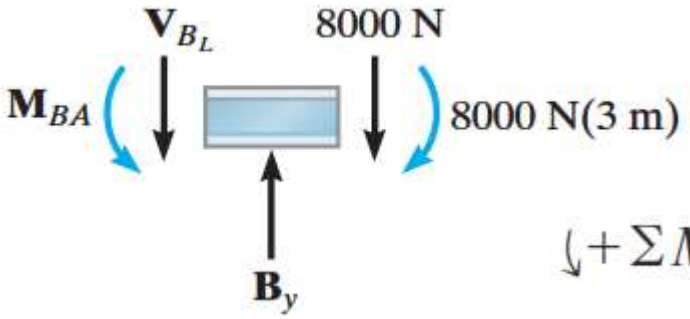
$$\psi_{AB} = \psi_{BA} = \frac{0.08\text{ m}}{4} = 0.02\text{ rad}$$

$$k = \frac{I}{L} = \frac{5(10^6)\text{ mm}^4(10^{-12})\text{ m}^4/\text{mm}^4}{4\text{ m}} = 1.25(10^{-6})\text{ m}^3$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2(200(10^9)\ \text{N/m}^2)[1.25(10^{-6})\ \text{m}^3][2(0) + \theta_B - 3(0.02)] + 0 \quad (1)$$

$$M_{BA} = 2(200(10^9)\ \text{N/m}^2)[1.25(10^{-6})\ \text{m}^3][2\theta_B + 0 - 3(0.02)] + 0 \quad (2)$$



$$\circlearrowleft + \Sigma M_B = 0; \qquad M_{BA} - 8000\ \text{N}(3\ \text{m}) = 0$$

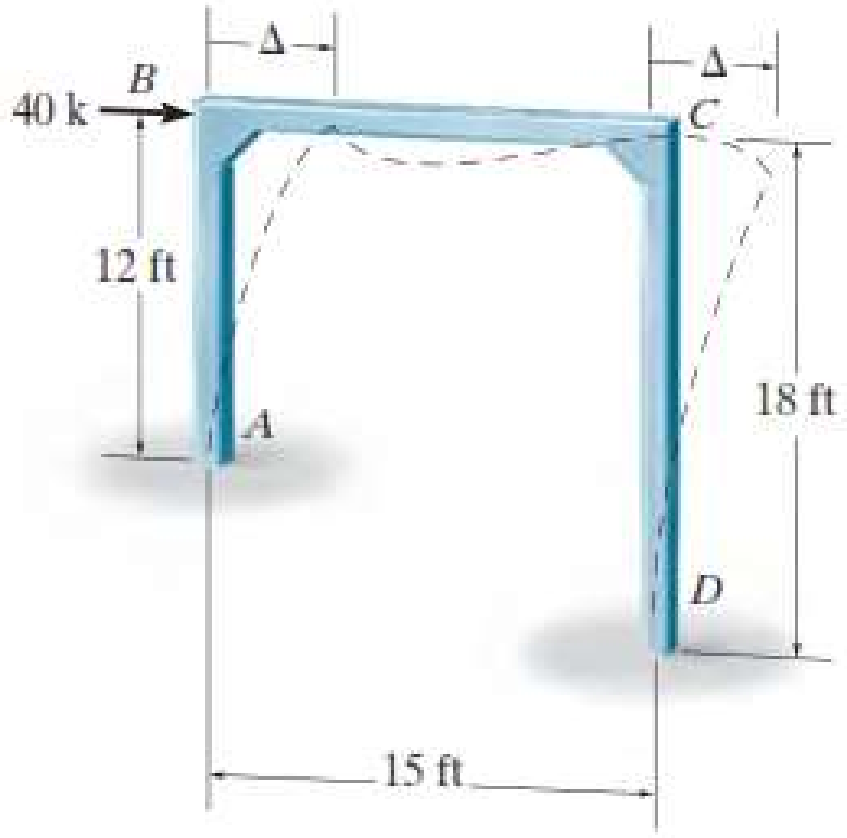
$$1(10^6)\theta_B - 30(10^3) = 24(10^3)$$

$$\theta_B = 0.054\ \text{rad}$$

$$M_{AB} = -3.00\ \text{kN} \cdot \text{m}$$

$$M_{BA} = 24.0\ \text{kN} \cdot \text{m}$$

Örnek 8: (Yanal Ötelemeli)



$$\psi_{AB} = \Delta/12 \qquad \psi_{DC} = \Delta/18. \quad \Rightarrow \quad \psi_{AB} = (18/12)\psi_{DC}$$

$$M_{AB} = 2E\left(\frac{I}{12}\right)\left[2(0) + \theta_B - 3\left(\frac{18}{12}\psi_{DC}\right)\right] + 0$$

$$= EI(0.1667\theta_B - 0.75\psi_{DC})$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)\left[2\theta_B + 0 - 3\left(\frac{18}{12}\psi_{DC}\right)\right] + 0$$

$$= EI(0.333\theta_B - 0.75\psi_{DC})$$

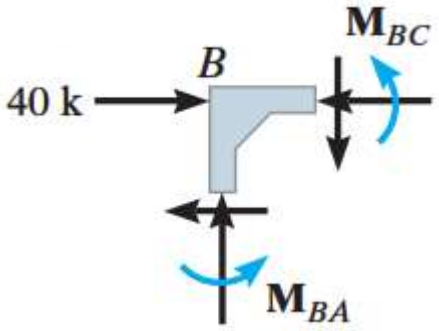
$$M_{BC} = 2E\left(\frac{I}{15}\right)[2\theta_B + \theta_C - 3(0)] + 0$$

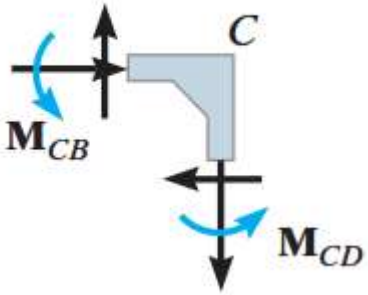
$$= EI(0.267\theta_B + 0.133\theta_C)$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)[2\theta_C + \theta_B - 3(0)] + 0 = EI(0.267\theta_C + 0.133\theta_B)$$

$$M_{CD} = 2E\left(\frac{I}{18}\right)[2\theta_C + 0 - 3\psi_{DC}] + 0 = EI(0.222\theta_C - 0.333\psi_{DC})$$

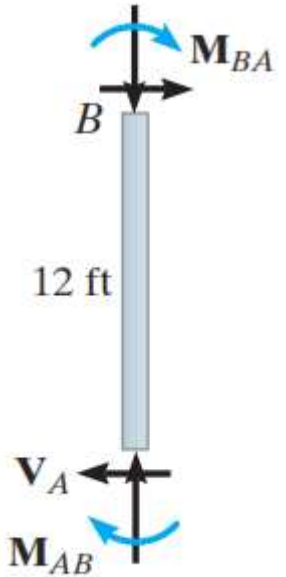
$$M_{DC} = 2E\left(\frac{I}{18}\right)[2(0) + \theta_C - 3\psi_{DC}] + 0 = EI(0.111\theta_C - 0.333\psi_{DC})$$

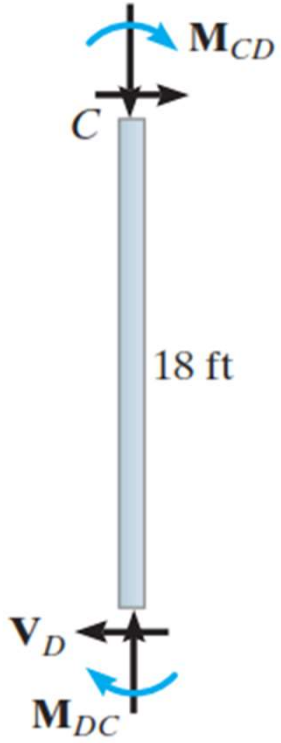




$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$





$$\pm\rightarrow \Sigma F_x = 0; \qquad 40 - V_A - V_D = 0$$

$$\Sigma M_B = 0; \qquad V_A = -\frac{M_{AB} + M_{BA}}{12}$$

$$\Sigma M_C = 0; \qquad V_D = -\frac{M_{DC} + M_{CD}}{18}$$

$$40 + \frac{M_{AB} + M_{BA}}{12} + \frac{M_{DC} + M_{CD}}{18} = 0$$

$$0.6\theta_B + 0.133\theta_C - 0.75\psi_{DC} = 0$$

$$0.133\theta_B + 0.489\theta_C - 0.333\psi_{DC} = 0$$

$$0.5\theta_B + 0.222\theta_C - 1.944\psi_{DC} = -\frac{480}{EI}$$

$$EI\theta_B = 438.81 \qquad EI\theta_C = 136.18$$

$$EI\psi_{DC} = 375.26$$

$$M_{AB} = -208 \text{ k} \cdot \text{ft}$$

$$M_{BA} = -135 \text{ k} \cdot \text{ft}$$

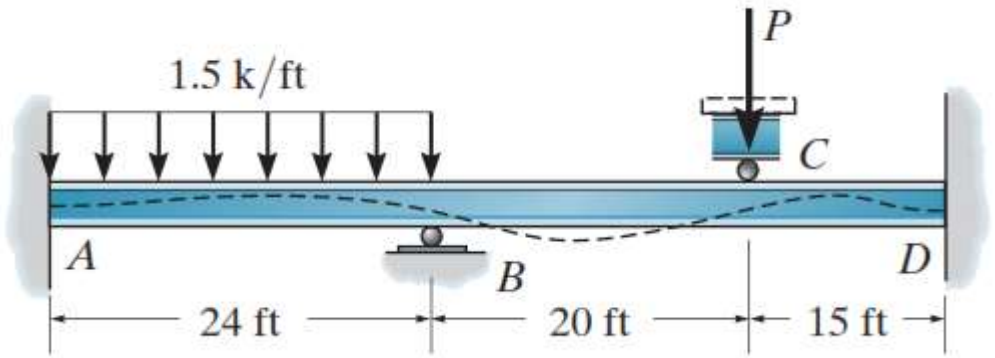
$$M_{BC} = 135 \text{ k} \cdot \text{ft}$$

$$M_{CB} = 94.8 \text{ k} \cdot \text{ft}$$

$$M_{CD} = -94.8 \text{ k} \cdot \text{ft}$$

$$M_{DC} = -110 \text{ k} \cdot \text{ft}$$

ÖDEV 6:



C noktası aşağıya doğru 0.1ft oturma yapıyorsa , şekilde gösterilmekte olan kirişin A ve B noktalarındaki momentini hesaplayınız.

I = 1500 in^4 E = 29(10^3) ksi.